

# 3-D analysis of gravity dams

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The traditional structural analysis of gravity dams, both concrete and fill, is based on an oversimplified two-dimensional scheme, which is largely misleading as soon as the valley slopes start to become steep. It is postulated that such dams should be analysed in a way that takes into account their real three-dimensional behaviour if unpleasant mishaps are to be avoided, such as have occurred at a number of concrete faced fill dams. A simplified way to study this problem is presented here, to get a first feeling of the various aspects related to the real conditions of equilibrium of gravity structures, especially in narrow valleys.

From the physical point of view, any dam represents a three-dimensional body and should thus be analysed taking into account this aspect of the reality [Lombardi, 1993<sup>1</sup>; 1994<sup>2</sup>; 2004<sup>3</sup>].

In fact, it can be noticed that, mainly for traditional reasons, straight gravity structures, (conventional or RCC gravity dams, but also fill dams with a core or a concrete face), are generally analysed as two-dimensional structures. That means that only the stability of thin 2-D cross-slices is investigated. The foundation surface for each slice is implicitly assumed to be horizontal in the direction of the dam axis, that is cross-valley. In other words, possible problems in the third dimension, that is, in the direction of the dam axis, or of the slope of the valley flanks, are simply overlooked or ignored (Fig. 1)

The historical reason for this way of thinking probably goes back to a period when the weak structural element was the concrete dam itself. The main scope of the analysis was thus to define the stresses in the dam body. Today, however, the weakest element is generally no longer the dam body itself, but its foundation. This change is the consequence of significant improvements in construction techniques, but also mainly the fact that the better dam sites have been already developed, and those now available are much less favourable as regards their geotechnical conditions.

The aim of the present paper is simply to show that, when the traditional approach is followed, risks may be assumed of which many designers are apparently not really aware. The reasons for focusing attention on the three-dimensional behaviour of the structure are manifold: risks are steadily increasing from decade to decade for various reasons. For example, the continuous technical and economical improvements of the RCC technique create a temptation to adopt this type of structure even where a fill dam could be the best solution, or even the only correct one from a technical and safety point of view.

Sometimes, even medium quality rock foundations are considered so as not to allow for the construction of an arch dam and thus to favour a gravity structure, giving the wrong impression that any foundation problem can then be automatically solved. Sometimes

such a structure is selected, instead of an arch dam, which could possibly be more appropriate for the geological and geotechnical conditions.

A number of powerful computation codes for gravity structures, frequently used without sufficient caution, may lead designers to put 'blind trust' in their results, with insufficient critical sensitivity or adequate cross-checking.

It is surprising to note that some quite fundamentally simple questions, such as the one to be discussed next, often escape the attention of designers. This can lead to dangerous situations [Bustamente and Tradisic, 2006<sup>4</sup>; Lombardi, 2006<sup>5</sup>].

## 1. Variable geotechnical conditions

According to Fig. 2, we may consider the simple case of a concrete gravity dam of constant height, built on ground of variable quality, that is, on rock zones of different shear strengths and deformabilities. This is a quite frequent occurrence. To compensate for possible differential settlements, as well as to take into account the contraction of the concrete by shrinkage and cooling, vertical joints are provided to form independent blocks, each one resting by itself on a homogeneous but different foundation.

The factor of safety (FoS) against sliding of each of the three blocks, considered as independent, thus differs; the lowest value of FoS may be sufficient or not, and the block itself may thus be sufficiently safe or not.

Should the decision then be taken to close the joints, to achieve an averaged overall factor of safety, thus to compensate for the too low local ones (or for any other reason), then the behaviour of the structure becomes three-dimensional: for the usual load cases (when the deformabilities are different); or just for the extreme ones (when the strengths are different), or at least when carrying out a limit equilibrium analysis.

To ignore these particular conditions may have costly and possibly even serious consequences.

Fig. 1. Traditional assumption for the analysis of a straight gravity dam. Independent 2-D slices across the dam body resting each one on a hypothetical horizontal foundation ( $W$  = weight;  $H$  = water pressure;  $U$  = uplift).

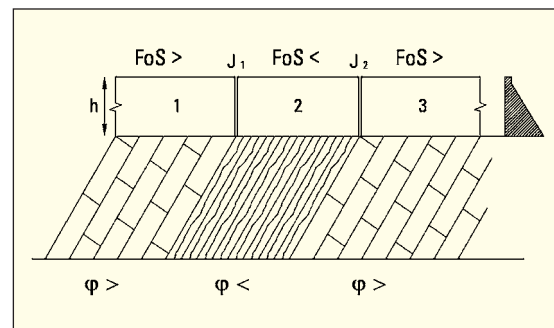
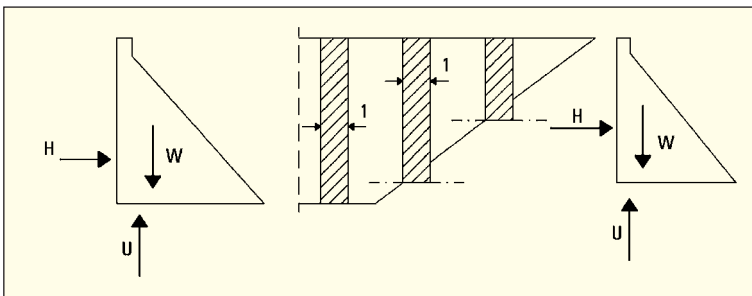


Fig. 2. Concrete gravity dam of constant height resting on a ground of variable quality. (FoS = factor of safety;  $\phi$  = friction angle), 1 to 3: block numbers;  $J_1, J_2$ : vertical joints.

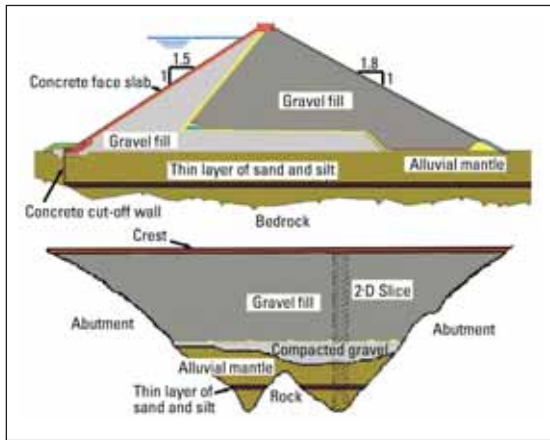


Fig. 3. CFRD dam. The imposed 2-D analysis for a 3-D problem led to additional useless expenses.

A real, but anonymous, case is shown by Fig. 3. It represents a quite large concrete faced rockfill dam resting on an exceptionally well compacted morainic material of the best quality. In a narrow irregular canyon below that material, a layer of fine sand was found, which could possibly liquefy in the event of a strong earthquake.

For anybody with sound geotechnical and structural judgement, the situation appears clearly to be a three dimensional one, as it is evident that a thin vertical slice cut in the massive dam cannot move out without pulling some part of the nearby volume of material, which of course would resist the movement and thus increase the factor of safety of the thin two-dimensional slice considered.

In spite of these simple considerations, a two-dimensional equilibrium analysis was specified and imposed on the owner. This led to the construction of an expensive but totally useless additional berm at the downstream dam heel.

This case is a very clear, but not unique, example of a 'simplification' of the problem. Unfortunately, some codes of analysis follow this way of thinking based on the same traditional 2-D considerations, as imposed at the project described.

## 2. The downhill stability

### 2.1 General aspects

The most important of the often overlooked factors in the analysis of gravity structures appears to be the reduced stability of the dam elements (or blocks) resting on the inclined valley flanks. It is therefore worthwhile to discuss this aspect and its influence on the equilibrium and on the factor of safety of the entire dam structure.

### 2.2 The single block on the slope

Let us first consider an independent concrete block resting on a valley flank. For the sake of simplicity, we will analyse only the case of sliding on the foundation surface, while possibly deeper, differently oriented discontinuities in the rock mass could obviously be more critical and should also be considered. To avoid additional useless complications at this stage, the foundation surface is simply assumed to be horizontal in the up/downstream direction.

According to Fig. 4, the foundation surface should resist the combined force  $R$  resulting from the vectorial composition of the downstream oriented hydrostatic pressure  $H$  (plus possibly a hydrodynamic

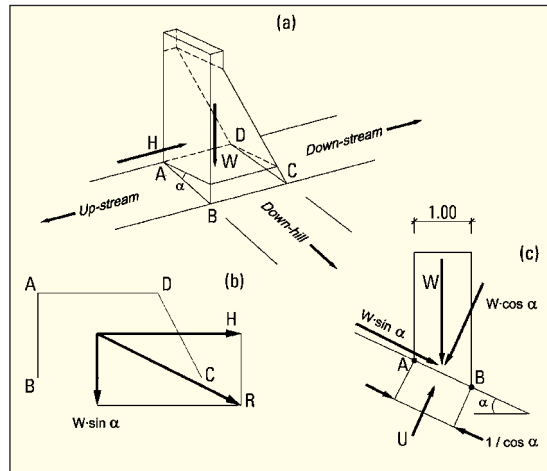


Fig. 4. Equilibrium of a single "independent" dam-block. (a) block on an inclined foundation; (b) foundation surface; and, (c) downhill cross section with main forces.  $\alpha$  = downhill inclination;  $W$  = weight;  $U$  = uplift;  $H$  = horizontal up/downstream driving force;  $R$  = resultant.  $\beta$  = multiplier of the uplift ( $\beta = 1$  for triangular uplift at no drains);  $n$  = required factor of safety;  $\varphi$  = corresponding friction angle.

effect) and of the downhill component of the weight ( $W \sin \alpha$ ), giving thus an 'inclined' action on the foundation surface.

On the other hand, the shear resistance available on the foundation surface depends on the force acting on it in the normal direction, while obviously taking into account the corresponding inclined uplift force.

Because of the inclination of the foundation, the 'uplift force' is increased in the proportion of  $1/\cos \alpha$ , corresponding to the actual width of the foundation in the downhill direction, with respect to the usual case. On the other hand, the weight component acting normally on the foundation is reduced in inverse proportion, thus being ( $W \cos \alpha$ ).

As can be seen in Fig. 5, the angle of friction theoretically required on the foundation for a given factor of safety is greatly increased as a function of the cross-valley inclination. There is no doubt that in many real cases, and for quite a number of existing dams, the concrete blocks on the valley flanks would not be stable by themselves, or at least would not show a sufficient factor of safety. The actual factor of safety, of course, stays well below the value obtained by the usual computations based on the assumption of a cross-valley horizontal foundation.

In many cases these blocks will need some support from the nearby ones, in fact, from the adjacent ones, on the way down to the central part of the valley, where presumably a horizontal foundation for the central blocks will exist, or where the compensation of the forces from the opposite valley flank may be found.

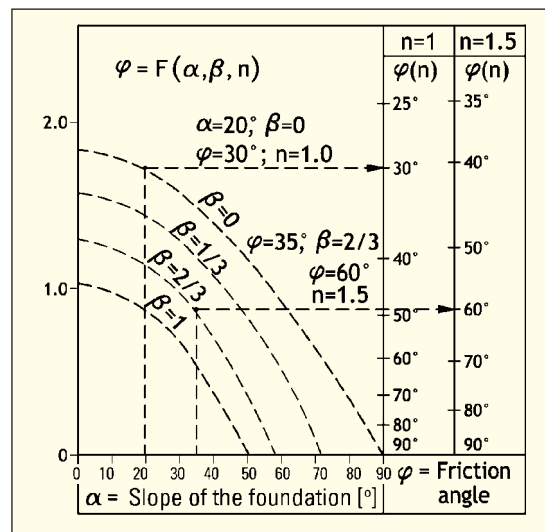
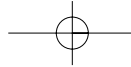


Fig. 5. The equilibrium of a block founded on an inclined surface. Only weight, water pressure and uplift are taken here into account. Example: if the cross valley slope is 35° and uplift equals 2/3 (of the triangular diagram), a friction angle  $\varphi$  of 60° would be required (at no cohesion) to obtain a factor of safety of 1.5 ( $\varphi = 49^\circ$  for a factor of safety of 1).



Clearly the situation becomes more critical, the steeper the valley flanks are.

It may be recalled that in former times it was quite usual to form the foundation of gravity dams by stair-like steps, so to place each single concrete block on a horizontal surface. The aim of this design was sometimes related to the old theoretical wrong vision dealt with here, but mainly the aim was to improve the stability of the single blocks at the time of construction.

However, such a design was based on a misinterpretation of the actual geotechnical situation, because inclined discontinuities paralleling the ground surface, which generally do exist, could easily cut off the tiers, and thus be a determinant for the stability. It should also be recalled that the geotechnical conditions were often better in the past than those found today. At present, the foundation line is thus, as a rule, shaped as an inclined smooth one, while the blocks are usually cast from the valley bottom up (especially for RCC dams).

### 2.3 Restoring the continuity of the dam body

As a result of the conditions described above, it is often unavoidable to restore the continuity of the structure in closing the joints and implementing special devices (like shear-keys) to increase the factor of safety by making possible a transfer of the required forces from the valley flanks down to the central part of the dam, which will supposedly resist them.

In this case, from the crest abutment down to the valley bottom, a number of factors may intervene which could increase the stability of the blocks and of the entire structure as, for example:

- the presence of downstream rock or concrete masses, which can at least partly resist the sliding forces of single blocks, and thus stabilize the entire wing of the dam; or,
- sections of the dam with a transversally flat foundation, which can interrupt the downhill flow of forces in resisting them directly; or,
- keys and specially shaped foundation surfaces with increased shear strength.

Such situations must obviously be taken into account in the design, but, for the sake of simplicity, are not be considered in the following presentation.

The analysis of the transfer of the forces, required to achieve a given factor of safety may be complex, as shown by Fig. 6. The three components X, Y and Z must obviously be taken into account at each step of the computation.

The way the forces will be transferred to the next block is influenced by the stiffness of the structures implemented in the joints, as well as by the rigidity of the foundation versus that of the structure itself.

Obviously, a complete 3-D finite element analysis of

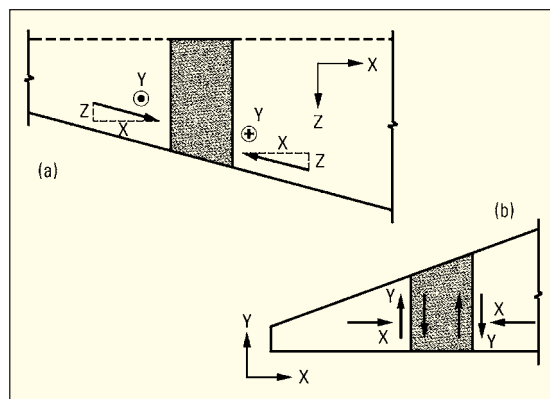


Fig. 6. Forces normally to be transferred by the joints from block to block. (a) Downhill cross-section. Transmission of compressive forces from block to block; (b) Plan view with the forces transferred from block to block through the joints

the whole structure can be carried out. This must, in any case, rely of a number of possibly arbitrary assumptions about various elements of the global structure. In addition, the shrinkage and the long-term cooling of the concrete may change the stress and stiffness conditions over the years, causing a modification to the stability conditions of the structure.

As the main aim of this paper is to define the overall conditions of equilibrium and of the factor of safety to be expected for the structure, rather than to compute the exact value of some local stresses (which might be analysed at a second stage), some simplified procedures can be considered. The later stress analysis can thus be confined to the points of stress concentration, which may appear, for example at the shear keys, as a result of the large forces to be transferred through the joints from one block to the next.

### 3. Simplified definition of the factor of safety

In the proposed simplified analysis, which represents at least a first step, the way to follow will be the so-called 'load factor method'. The actions, like water pressure, uplift, silt load, earthquake (for example, considered as pseudo-static), will be multiplied by the required factor of safety, while the resisting forces (mainly the self weight, the friction and the cohesion on the foundation) will be considered with their actual values.

Another way could obviously be to keep all the loads at their actual value, but reduce the geotechnical strength factors, dividing them by the required factor of safety. The reason for doing this could be the intention to obtain a clearer definition of the points of impact of the forces.

The method of partial factors of safety could also be used. This is in fact a combination of the two methods mentioned above. For the sake of simplicity, the 'load factor' method is used here.

The absolute value of the resisting shear force along the foundation surface can quite easily be defined on the basis of the friction angle and the cohesion assumed on surface. On the contrary, its direction is not known in advance, and depends on a number of factors.

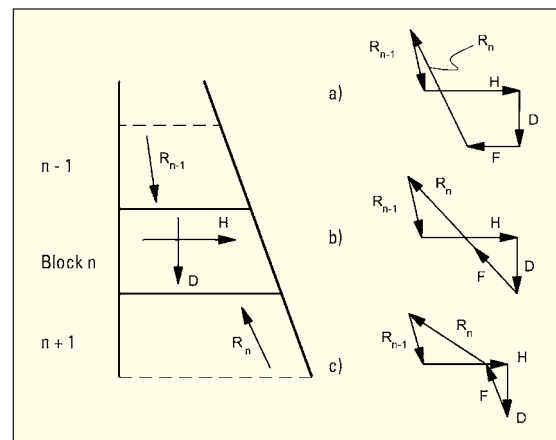
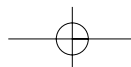


Fig. 7. Foundation surface. Three typical cases for the equilibrium of a single block.  $R_{n-1}$  = action from the uphill block;  $H$  = horizontal driving force;  $D$  = downhill component of the weight;  $F$  = resistance by friction and cohesion on the foundation; and,  $R_n$  = reaction by the downhill block. Obviously the vertical components are considered accordingly (a) first limit, (b) intermediate case, (c) second limit.



As regards the direction of these shear forces on the foundation, two limits as well as intermediate values may be considered, as shown at Fig. 7.

- *The first case - Fig. 7(a)*

It is assumed that, for each block, the downhill component will be directly supported by the next block by a compressive force. The frictional resistance of the foundations is thus entirely devoted to resisting the downstream component, that is the driving force  $H$ . If this resistance were insufficient, the difference is then supposed to be transferred to the next downhill block by a transversal shear force acting in the joint.

- *The second case - Fig. 7(b)*

It is assumed that, for any block, the shear resistance of the foundation is oriented in the opposite direction to the resultant of the forces acting directly on it. Again, if required, to restore the equilibrium, the lacking forces will be transferred to the next block by the joint.

- *Other assumptions - Fig. 7(c)*

These can be assumed, depending on the nature and configuration of the joints, for example, if the joints are 'open' in their normal direction, the shear resistance of the foundation will act, in the first place, to support the downhill component of the forces. If necessary the shear and even the compressive resistance of the joint must intervene to ensure the downstream stability of the block with the required safety factor.

Obviously the computation is carried out, in all cases, from the crest elevation down to the valley bottom, and each block is analysed taking into account the forces coming down from the upper ones.

For the sake of the computation, additional 'virtual' joints may be introduced where a change in the conditions is given, as for example, if the assumed values for the friction angle or the cohesion, as well as the inclination of the foundation from a section to the next one will change.

It appears that, in general, the first case considered above will be excessively favourable, while the second one, which may look more logical, is somewhat detrimental to the equilibrium. It is felt that the most probable solution will be in between these limits. It should therefore be worthwhile to fulfil the computations on the basis of various assumptions, even in the case of a finite element analysis, so as to obtain a better and more complete overview of the situation.

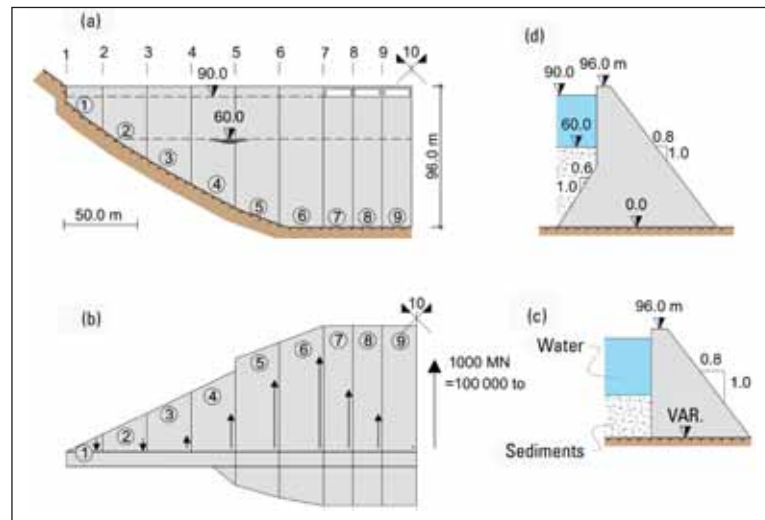
Indeed, the choice between the various assumptions about the direction of the shear on the foundation surface depends on a number of factors, but first and foremost on the nature of the joints, their opening, as well their relative stiffness and strength both in the radial and the tangential directions. In an actual dam, various additional assumptions may apply for the various blocks.

It goes without saying that, at the end of the day, the central part of the dam, at the valley bottom, should be able to resist, at least with the required factor of safety, the forces coming down from both valley flanks.

#### 4. Geotechnical parameters

The question of the geotechnical parameters to be selected as a basis for the computations is obviously of the greatest importance.

In particular, it should be considered whether a displacement by sliding takes place along the foundation or along any somewhat deeper discontinuity in the rock mass. In this case, the corresponding geotechnical values should possibly be used, for simplicity, at the foundation surface itself.



In case cohesion is taken into account, for example, according to the Mohr-Colomb law, it should also be considered that possibly even a small sliding movement which could be caused by a seismic event, could destroy it. Therefore, pre- as well as post-sliding conditions must be analysed.

The question of the geotechnical parameters is particularly important because in many cases the friction angle to be considered may be only slightly greater than the slope of the foundation surface.

#### 5. An example

Following the concepts presented above, a real case was computed and the result are given in Fig. 8. This shows the main data of the left flank of an RCC dam 96 m high, with its actual joints, with the assumption that the joints cannot transfer normal forces but only shear forces.

It is assumed that the shear forces are transferred from block to block at a short distance above the foundation line. The results of the computation are represented in the Figure as well as by Table 1. These results call for two main comments.

First, the overall factor of safety is only 1.5, while the 2-D analysis showed a value of 1.85 for a horizontal foundation (Blocks 7 to 9). Second, it should be mentioned that the forces to be transferred through the joints are very significant, and therefore adequate devices must be designed to resist them. In addition, it should be noted that the factor of safety depends greatly on the dimensions of the block on the valley bottom. For example, if block 9 did not exist, then the factor of safety would be about 1.4 instead of 1.5.

#### 6. Summary and conclusions

According to a quite old tradition, gravity dams (including fill dams) are generally computed as two-dimensional structures, with a study only of cross sections normal to the dam axis; usually even only the highest one, at the centreline of the valley bottom is analysed.

It must be considered that in reality a gravity dam exhibits three-dimensional behaviour as soon as the height of the structure is variable along its axis or as soon the geotechnical conditions are not uniform all along the foundation line.

It is thus postulated that instead of continuing with traditional habits, these dams should be analysed taking into account the real conditions, that is the actual three-dimensional behaviour of the structure.

A simplified calculation method has been presented

Fig. 8. Example of a dam. (a) view from upstream; (b) plan view; (c) cross-section lateral blocks 1 to 6; (d) cross-section central blocks 5 to 9. Load case: normal water elevation 90 m, sediments to 60 m. In the plan view (b) the shear forces in the joints are shown. They are required to obtain a safety factor of 1.5, in the case the joints cannot resist normal forces. These shear forces reach 970 MN (97 000 tonne).

here as a first step in doing this, but here, as a first step, it is mainly intended to raise awareness.

The actual state of equilibrium, of the various possible assumptions shown, depends strongly on the nature of the vertical joints and on their stiffness in the normal and tangential directions. More attention should be devoted to this problem than is usually done, at least to avoid unpleasant situations.

The forces to be transferred from one block to the next to ensure a sufficient factor of safety can be extremely high and adequate provisions must be taken if significant mishaps are to be avoided.

Similar conditions may be observed also for fill dams and some recent incidents which have occurred at concrete faced rockfill dams also appear to be related to 3-D conditions of equilibrium.

Finally, it can be said that in fact gravity dams, both concrete and fill ones, were originally (at least implicitly) conceived for wide valleys with gently sloping flanks, that is, for a slowly changing height.

The development of some construction techniques is leading to the implementation of this type of dam also on steep flanks, that is, in narrow valleys.

Often the consequences of this evolution are overlooked and this could have serious consequences, for example in the event of heavy earthquakes.

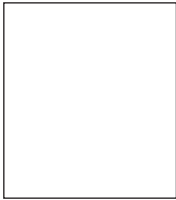
The question dealt with here is quite important, because the actual factor of safety is often significantly smaller than may be believed on the base of the usual two-dimensional analysis. This problem is not limited to the question of structural safety, but may also have an economic impact and may thus lead to different conclusions in the choice of dam type, especially if the fact is taken into account that the available

dam sites are becoming ever more challenging from the geotechnical point of view.

Finally, it should be recalled that sliding on the foundation surface is only one of many possible failure modes. In particular, sliding on deeper discontinuities in the foundation mass must be duly taken into account, as well as other failure modes such as toppling or overstressing of the shear keys in the joints. ♦

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