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# COMPARISON OF FORMULAE FOR THE PREDICTION OF SCOUR DEPTH AT PIERS

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## ABSTRACT

Local scour at bridge piers was extensively studied in the literature and several empirical formulae were proposed to assess the maximum scour depth at equilibrium. Since the equations were derived in small scale conditions, the application to practical cases is uncertain. Recently, formulae were tested by using field datasets, but in few cases only. In this paper, six design formulae for the estimation of the equilibrium maximum scour depth at a circular pier were compared by using synthetic and original field data. In the former case, random input variables were generated with the Monte Carlo simulation technique in realistic ranges typical of many natural watercourses, in both clear-water and live-bed scour conditions, in order to achieve useful results for field applications. The results show that different formulae produce significantly different predictions. The selected formulae were also tested on a field dataset in the case of uniform bed sediments, confirming that their ability to assess the maximum scour depth at equilibrium is not satisfactory.

## INTRODUCTION

Over the past 50 years, numerous equations have been developed to predict local scour depth at bridge piers. Most formulae are based on laboratory experiments, and only some of them have already been tested using field data. The limited use of field datasets leads to some uncertainty in the application of pier-scour formulae to predict scour in real cases. Based on a brief consideration of the literature, Johnson (1995) used field data to consider the accuracy and limitations of seven pier-scour formulae in various field conditions. An extensive analysis and comparison of field data with predictions of five selected formulae by Landers and Mueller (1996) showed that none of the selected formulae accurately estimates the depth of scour in all the measured conditions. Their results also clarified that some of the equations performed well as conservative design formula; however they overestimated many observed scour depths by large amounts. The results of a more recent study by Mohamed et al. (2006) showed that the CSU formula, which was later improved and published as FHWA (HEC-18) formula (Richardson and Davis, 2001), is more accurate in comparison to other three selected pier-scour formulae.

A review of previous studies shows that a large number of empirical formulae was proposed to estimate pier scour depth. All proposed equations can be classified in two groups: 1) formulae developed through a correlation among effective parameters and scour depth; 2) formulae based on envelope curves, in order to estimate the maximum scour depth for safe design purposes. In this study, six well-known pier scour formulae were selected for comparison. All of the selected formulae were recommended in literature for design procedure. Owing to the general lack of field datasets, a comparison between formulae (two by two) can be carried out using large synthetic datasets, which can be generated in various hydraulic and sediment conditions. The main purpose of the present work is to compare the performance of six bridge pier scour equations using both randomly generated and field data.

## SELECTED FORMULAE

The selected formulae are presented below.

a) Breusers et al. (1977) formula (hereinafter BR) for clear-water and live-bed scour conditions:

$$\frac{d_{se}}{b} = 2\left(2\frac{U}{U_c} - 1\right)\tanh\left(\frac{h}{b}\right)\cdot K_s \cdot K_\theta \tag{1}$$

where $d_{se}$ is the maximum scour depth in equilibrium condition, $b$ the pier width, $U$ the approaching flow velocity, $U_c$ the critical velocity for sediment motion – computed with the Neill (1973) equation in SI units ($U_c = 31.08\theta^{1/2}h^{1/6}d_{50}^{1/3}$), where the Shields mobility parameter, $\theta$, can be computed based on sediment size (see Mueller and Wagner 2005, p. 20) – $h$ the approaching flow depth, $K_s$ the pier shape factor and $K_\theta$ the pier alignment factor (see details in Breusers et al., 1977).

b) Jain and Fischer (1979) formula (hereinafter JF) for clear-water and live-bed scour conditions:

$d_{se} = 1.84\,b\left(\frac{h}{b}\right)^{0.3} Fr_c^{0.25}$, valid for $Fr\text{-}Fr_c<0$ in clear-water conditions; (2a)

$$d_{se} = 2.0\,b\left(\frac{h}{b}\right)^{0.5}\left(Fr - Fr_c\right)^{0.25}$$, valid for $Fr\text{-}Fr_c \geq 0.2$ in live-bed conditions (2b)

where $Fr=U/(gh)^{0.5}$ and $Fr_c=U_c/(gh)^{0.5}$ are the Froude number and the critical Froude number, respectively. For $0<Fr\text{-}Fr_c<0.2$ the largest value which is obtained from Eqs. (2a,b) is to be taken.

c) Froehlich (1988) formula (hereinafter FL) for live-bed scour condition:

$$d_{se} = 0.32\,b\,\phi\,Fr^{0.2}\left(\frac{b_e}{b}\right)^{0.62}\left(\frac{h}{b}\right)^{0.46}\left(\frac{b}{d_{50}}\right)^{0.08} \tag{3}$$

where $b_e$ is the width of the bridge pier projected orthogonally to the approach flow, $\phi$ a coefficient based on the shape of the pier nose and $d_{50}$ the median grain size.

d) Kothyari et al. (1992) formula (hereinafter KR) for clear-water scour condition:

$$\frac{d_{se}}{b} = 1.0\left(\frac{b}{d_{50}}\right)^{-0.25}\left(\frac{h}{d_{50}}\right)^{0.16}\left(\frac{U^2 - U_{cp}^2}{\frac{\Delta\gamma_s}{\rho_f}d_{50}}\right)^{0.4}\alpha^{-0.3}\,,\quad U_{cp}^2 = 1.2\left(\frac{\Delta\gamma_s}{\rho_f}d_{50}\right)\left(\frac{b}{d_{50}}\right)^{-0.11}\left(\frac{h}{d_{50}}\right)^{0.16}\,,\quad \Delta\gamma_s = \gamma_s - \gamma_f \tag{4}$$

where $U_{cp}$ is the critical velocity for the motion of sediment particles at the pier nose, $\gamma_s$ the sediment specific weight, $\gamma_f$ the fluid specific weight, $\rho_f$ the fluid mass density and $\alpha=(B\text{-}b)/B$ the opening ratio, $B$ being the flume width or centre-to-centre spacing between two piers. In this study, $B$ was assumed to be much greater than $b$ and, therefore, its effect was neglected.

e) Melville (1997) formula (hereinafter ML) for clear-water and live-bed scour conditions:

$$d_{se} = K_{hb}K_I K_d K_S K_\theta K_G \tag{5}$$

where $K_{hb}$, $K_I$, $K_d$, $K_S$, $K_\theta$ and $K_G$ are coefficients taking into account the depth scale, the flow intensity, the sediment size, pier shape, pier alignment and channel geometry effects on scour depth, respectively; $K_{hb}$, $K_I$ and $K_d$, can be calculated as follows:

$$K_{hb} = \begin{cases} 2.4b & for \quad b/h < 0.7 \\ 2\sqrt{hb} & for \quad 0.7 < b/h < 5.0 \\ 4.5h & for \quad b/h > 5.0 \end{cases} \qquad K_I = \begin{cases} \frac{U}{U_c} & for \quad \frac{U}{U_c} < 1 \\ 1 & for \quad \frac{U}{U_c} \geq 1 \end{cases} \qquad K_d = \begin{cases} 0.57\log\left(2.24\frac{b}{d_{50}}\right) & for \quad \frac{b}{d_{50}} \leq 25 \\ 1 & for \quad \frac{b}{d_{50}} > 25 \end{cases} \tag{6}$$

Factors $K_S$ and $K_\theta$ can be also calculated by using the tables proposed in Melville (1997). In addition, $K_G=1$ for piers.

f) FHWA (HEC-18) formula (Richardson and Davis, 2001) (hereinafter HC) for clear-water and live-bed scour conditions:

$$\frac{d_{se}}{b} = 2K_1K_2K_3K_4K_w\left(\frac{b}{h}\right)^{0.35}Fr^{0.43} \tag{7}$$

where $K_1$, $K_2$, $K_3$, $K_4$ and $K_w$ are correction factors accounting for pier nose shape, flow angle of attack, presence of bed forms, bed armouring and wide piers in shallow flows, respectively (see Richardson and Davis, 2001, for details).

## COMPARISON BETWEEN FORMULAE BASED ON RANDOMLY GENERATED DATA

### Data generation

The following conditions were assumed in the data generation procedure:

1) the pier shape is circular;
2) the bed materials are incoherent uniform sediments with geometric standard deviation of particle size distribution $\sigma_g<1.5$ and relative submerged sediment density $\Delta=1.65$;
3) flow regime is assumed to be uniform (the riverbed slope is equal to the friction slope: $S=S_f$) in clear-water ($0.5\leq U/U_c<1$) and at the inception of sediment motion or in live-bed ($U/U_c\geq 1$) scour conditions, in a wide channel ($R_h \approx h$), $R_h$ being the hydraulic radius;

4) the well-known Manning and Strickler equations (respectively $U = R^{2/3}S^{0.5}/n$ and $n = 0.041d_{50}^{1/6}$, $n$ being the Manning roughness coefficient) were used to calculate the approaching flow velocity; $U_c$ was estimated through the Neill (1973) equation.

By using Monte Carlo techniques, 10,000 triplet input data ($h$, $b$, $d_{50}$) were synthetically generated for clear-water conditions and 10,000 data for live-bed conditions in the following ranges of values, which are typical of many natural watercourses: $0.5 \le h \le 10$ m, $0.5 \le b \le 5$ m and $0.062 \le d_{50} \le 64$ mm, discarding unrealistic combinations of the variables. Hence, the Manning roughness coefficient, $n$, was computed with the Strickler formula and $U_c$ with the Neill (1973) equation. Afterwards, 10,000 values of $U$ were randomly generated in order to verify the condition $0.5 \le U/U_c < 1$ for clear-water scour, and other 10,000 values in order to have $U/U_c \ge 1$ for the inception of sediment motion or live-bed scour conditions. Finally, the corresponding values of bed slope, $S$, were computed through the Manning formula, verifying that they were compatible with realistic cases (from $4.04 \cdot 10^{-5}$% to 0.68% in clear-water condition; from 0.0034% to 1% in live-bed condition). In addition, $\sigma_g$ was generated in the range 1 to 1.5 for uniform bed sediments. Assuming a normal distribution of sediment size, the characteristic sediment size which is required in Eq. (7) was calculated as $d_{95} = d_{50}\,\sigma_g^{1.645}$. The bed form factor in HEC-18 formula was assumed to be $K_3$=1.1 (as recommended for dune height less than 3 m in live-bed scour condition). For each pair of formulae, 10,000 values of $d_{se}$ were calculated and compared. The number of comparisons between $m$ formulae taken two by two is equal to $m(m-1)/2$. Among the 6 considered formulae, 4 were developed for both clear-water and live-bed scour conditions (BR, JF, ML and HC), 1 for clear-water scour condition (KR) and 1 for live-bed scour condition (FL). Therefore, 10 comparisons in clear-water scour condition and 10 comparisons in live-bed scour condition were carried out, for a total of 20 comparisons.

## Result analysis

Figure 1 shows some examples of the matching between the output of formulae taken two by two, with the perfect agreement lines and the ±10% bands. The percentage of simulated points located below, inside and over the band is presented in Tables 1 and 2 for clear-water and live-bed conditions, respectively. For example, Table 1 shows that HC formula in 44%, 55% and 1% of simulated conditions furnishes respectively values smaller than, almost equal to, and higher than those given by the JF formula in the same condition. In other words, in few conditions HC formula may give higher prediction with respect to JF formula (see also Figure 1a). The agreement between formulae A and B can be considered as “strong”, “moderate” or “weak” if 75 to 100%, 50 to 75% and 0 to 50% of simulated points fall inside the ±10% band, respectively.

In clear-water condition, Table 1 shows that a strong agreement between two formulae is never found. Only the JF formula is in moderate agreement with the HC formula, with only 55% of simulated data falling inside the ±10% band (Figure 1a). The other formulae show weak agreements with all the other ones (see, e.g., Figure 1b). The JF formula produced overestimations with respect to the other formulae in many simulated conditions (see, e.g., Figure 1c,d), due to the fact that it estimates the maximum clear-water scour depth for $U \approx U_c$.

In live-bed scour condition, no strong nor moderate agreements were found. The best (but weak) agreement was obtained between FL and BR formulae (42% of points falls inside the ±10% band; see Table 2). However, as shown in Figure 1e, the FL formula may predict scour depths significantly higher than BR formula in some conditions. Lower agreements may be expected between the JF and HC or ML formulae. Even if sometimes the output scatter was not very high, as in the case of the comparison between the ML and BR formulae (Figure 1f), the agreements were very weak.

**Table 1. Percentage of points falling below, inside and over the ±10% bands in clear-water condition**

| | | *B* | | | |
|---|---|---|---|---|---|
| | | *HC* | *ML* | *KR* | *JF* |
| *A* | *BR* | 11, 24, 65 | 0, 3, 97 | 2,37,61 | 4, 21, 75 |
| | *JF* | 44, 55, 1 | 41, 26, 33 | 50,30,20 | - |
| | *KR* | 26,31,43 | 15,30,55 | - | - |
| | *ML* | 43, 28, 29 | - | - | - |

**Table 2. Percentage of points falling below, inside and over the ±10% bands in live-bed condition**

| | | *B* | | | |
|---|---|---|---|---|---|
| | | *HC* | *ML* | *FL* | *JF* |
| *A* | *BR* | 0, 2, 98 | 0, 4, 96 | 34, 42, 24 | 1, 8, 91 |
| | *JF* | 5, 32, 63 | 53, 26, 21 | 82, 10, 8 | - |
| | *FL* | 2, 3, 95 | 6, 13, 81 | - | - |
| | *ML* | 5, 12, 83 | - | - | - |

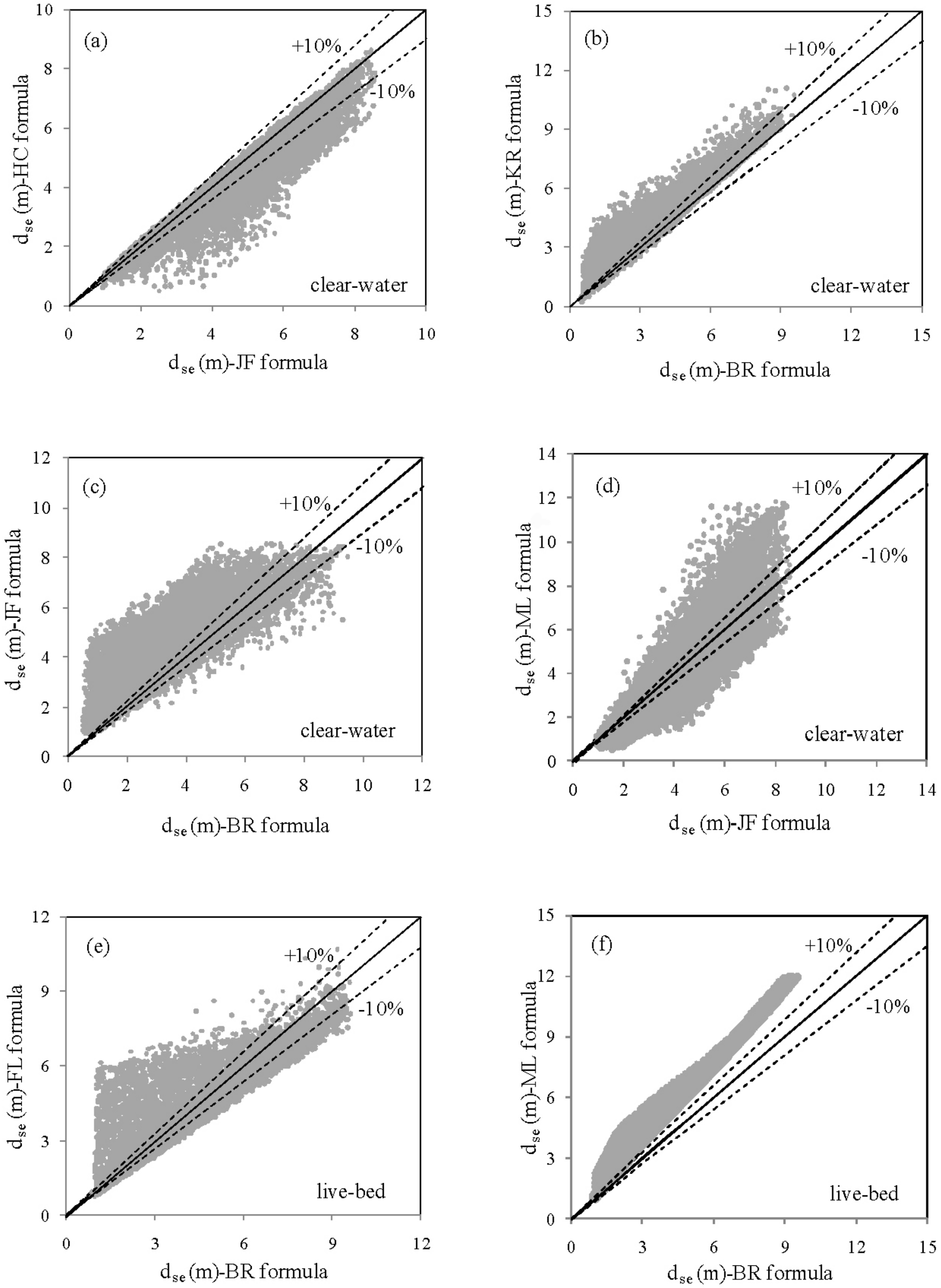

**Figure 1. Comparison of scour depths predicted by formulae taken two by two (formula A on the abscissa)**

## COMPARISON OF THE FORMULAE BASED ON FIELD DATA

The predictions of the selected formulae were also compared with available field data for 29 bridge piers in uniform bed sediments ($\sigma_g$<1.5). Data was selected from the whole dataset by Mueller and Wagner (2005), which was collected at 79 river sites in the USA. In fact, two aspects of each formula should be evaluated: 1) owing to safety problems, the predicted values of each formula needs to be around or higher than the observed field values; 2) owing to economical problems, the formulae should not overestimate excessively. Based on these two considerations, an asymmetric band was defined to evaluate the performance of the selected formulae. The upper and lower limits of the band were assumed to be +50% and -10%. In other words, a maximum safety factor equal to 1.5 was accepted for economical reasons, whereas a minimum underestimation factor equal to 0.9 was accepted for safety reasons. Table 3 shows the percentage of points falling inside the asymmetric bands. Although all the formulae do not perform satisfactorily, in clear-water scour condition the HC formula predicts the scour depth better than the other ones (see Figures 2a). Both overestimated and underestimated values were obtained by applying the BR formula (see Figure 2b). The ML, JF and KR formulae predicted the scour depths significantly higher than the observed values (see, e.g., Figures 2c,d). Similar result was reported by Grimaldi et al. (2006) comparing predictions of the ML formula with laboratory data.

In live-bed scour conditions, more overestimations were obtained by using the selected equations. Among them, the HC and FL formulae predicted the scour depth better than the other ones; however, in many conditions they significantly overestimated the scour depths (see Figures 3a,b). With the other selected formulae, the scour depths in live-bed scour conditions are even more overestimated (see, e.g., Figures 3c,d).

**Table 3. Percentage of points falling inside the asymmetric bands**

| *Scour condition* | *BR* | *JF* | *FL* | *KR* | *ML* | *HC* |
|---|---|---|---|---|---|---|
| Clear-water | 14 | 0 | - | 14 | 0 | 43 |
| Live-bed | 18 | 14 | 23 | - | 9 | 23 |

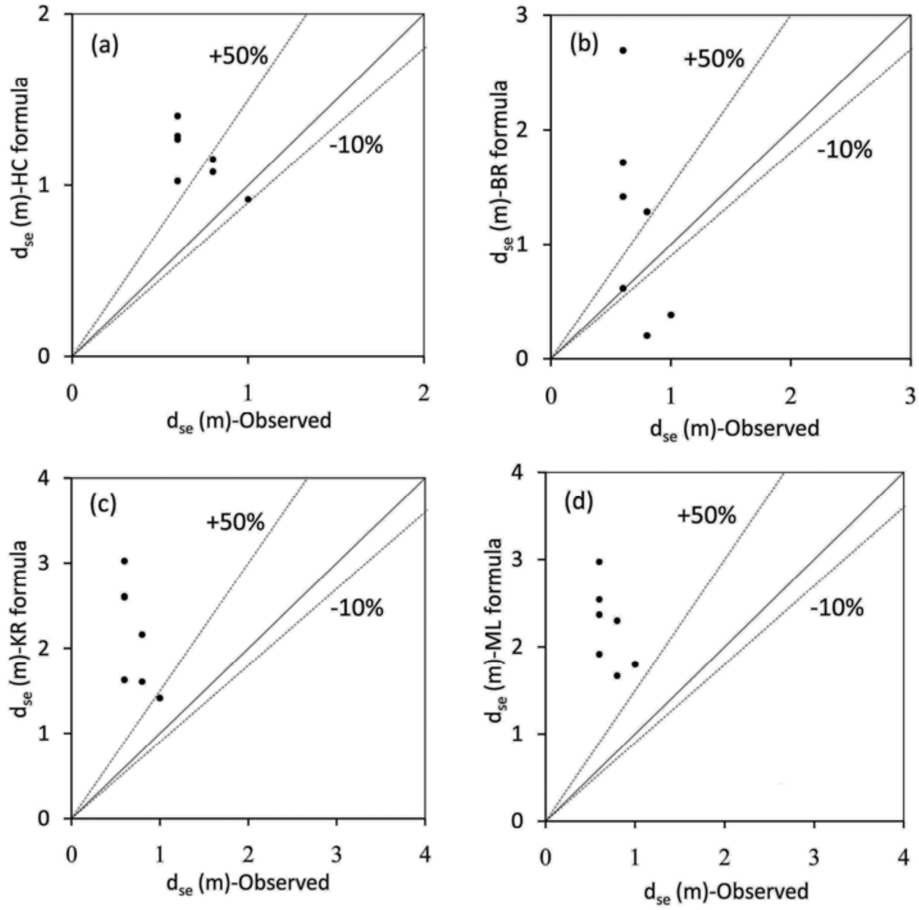

**Figure 2. Comparison of predicted and observed values of the scour depths in clear-water condition**

## CONCLUSIONS

Six well-known design formulae for predicting the maximum equilibrium scour depths at bridge piers in clear-water or live-bed conditions were compared two by two, by using synthetic randomly generated data. The results show that in most conditions the agreement between formulae is weak, being good at best in 55% of the analysed cases.

The formulae were also tested on a field dataset available in the literature for uniform sediments, producing unsatisfactory results. The HC formula in both clear-water and live-bed scour and the FL formula in live-bed condition predicted the measured scour depths better than the other selected formula.

This study evidences that none of the selected formulae accurately predicts the maximum equilibrium scour depths in the field and that high result variations are often obtained by using different pier scour formulae. Further research is needed to give a more comprehensive evaluation of pier scour formulae.

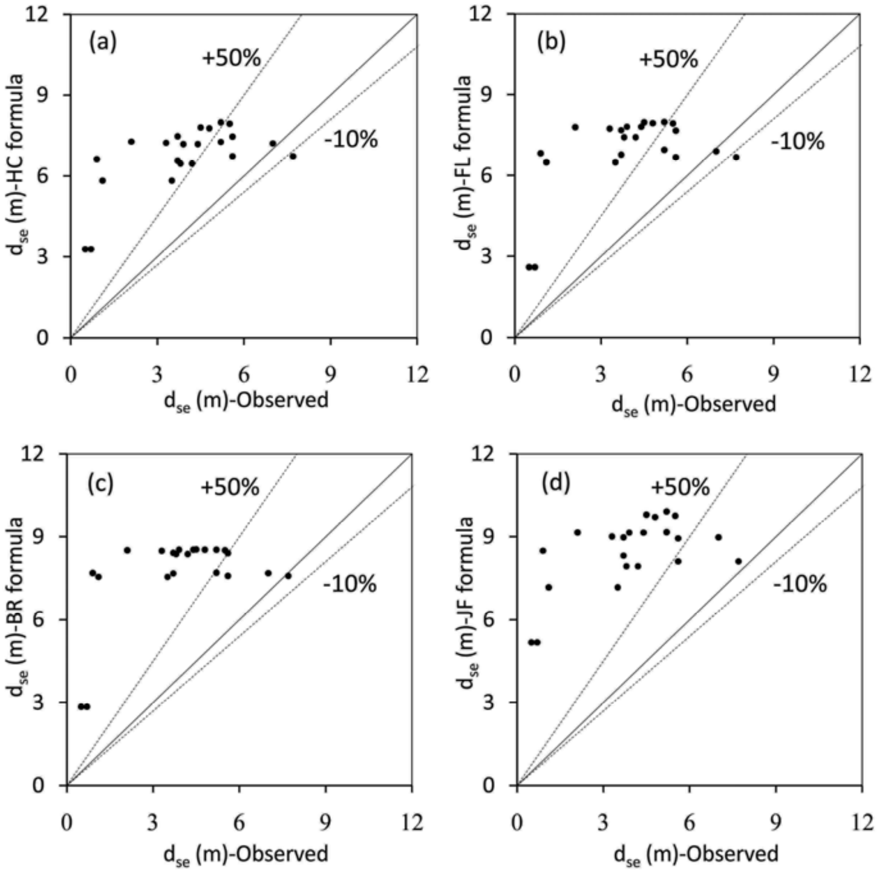

**Figure 3. Comparison of predicted and observed values of the scour depths in live-bed condition**

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