102.1-R-242 / VII.00092.01-EN

For a correct interpretation of ground reaction curves

F. Amberg
Lombardi Engineering Ltd., Minusio, Switzerland

12th Congress ISRM Beijing China, 16-21.10.2011

ABSTRACT: After a brief review of the convergence-confinement method, the paper presents some possible applications of the ground reaction curves showing its usefulness for the analysis of underground excavation problems. In fact a ground reaction curve can be defined by means of numerical analyses for any type of geometry and loading condition. Subsequently, the concept of the ground reaction curve as lower boundary for a region of possible equilibria is also presented and discussed. Due to the fact that the ground reaction curve is determined by a progressive reduction of the support pressure, the stress state within the failure zone around the tunnel is at failure limit. However, in case of an increasing support pressure after lining placement, a recompression of the ground in radial direction can be observed. In these cases the final equilibrium will be above the ground reaction curve.

SUBJECT: Analysis technique and design methods

KEYWORDS: Rock support, Numerical modeling, Tunneling

1 INTRODUCTION

Sometimes the term ground reaction curve (GRC), or characteristic line, is associated to a calculation method, probably even referring to an old and obsolete calculation method. This association is a clear misunderstanding. In fact, for a given underground excavation geometry, the ground reaction curve represents the displacement of a selected point on the excavation boundary as a function of the support pressure. To obtain said curve it is necessary to perform a series of equilibrium calculations, which can be carried out in using any desired computation method.

The classical method refers to the simple case of axial symmetry in an infinite elasto-plastic medium. The behavior of a tunnel excavation in such conditions dates back to Fenner (1938). Subsequently, numerical solutions have been developed allowing to considerably enhance the possibilities offered by these analyses (Lombardi 1966).

Today, where numerical models are based on finite element or finite difference methods, it is quite simple to include any desired effect in the analysis, e.g. any shape of the cross section, the tunnel face, the presence of a fault beside the tunnel, a shallow tunnel, the gravity, the creep, the swelling, the schistosity, the stratification of the rock or the anisotropy of the initial stress state.

The great advantage of the ground reaction curve is that the result is not limited to a single equilibrium, but that it allows immediately to verify how the equilibrium of the excavation might change for different support conditions. This is an essential aspect for analyzing and optimizing the design of a tunnel in case of convergences. It facilitates the dimensioning of the tunnel lining with respect to the construction procedure and allows the assessment of the safety conditions of the excavation.

The determination of the rock load acting on a support (lining) is a very complex task: it involves the practically unknown rock mass properties, the construction method and the actual site conditions, which in a model are strongly simplified. It is also important to realize, that for every computation, the final equilibrium obtained in the analysis depends on a series of assumptions and more or less arbitrary choices and is never the result simply obtained with the calculation. By means of 3D-simulations it might be possible to refine the determination of the final load on the support, even if the simulation of the actual support behavior (increasing stiffness of shotcrete, gap between steel ribs and rock mass, settlement at ribs foot, etc.) is quite complex so that its simplification needs also a series of assumptions.

Especially, because of this complexity the ground reaction curve becomes a very useful technique that should never be ignored in the design of an underground excavation.

2 EXAMPLES OF GROUND REACTION CURVES

2.1 *Roof stability of a shallow tunnel*

One of the classical methods to analyze the bearing loading acting on the support of a shallow tunnel refers to the model proposed by Terzaghi (1943). The formulation is given by Equation 1 with notations according to Figure 1.

$$\sigma_v = \frac{b \cdot \gamma - c}{\lambda \cdot tan\varphi}\left(1 - e^{-\frac{h}{b} \cdot \lambda \cdot tan\varphi}\right) \quad (1)$$

Being φ and c the Coulomb's strength parameters, γ the unit weight and λ the ratio between vertical and horizontal stresses within the silo (coefficient of lateral earth pressure).

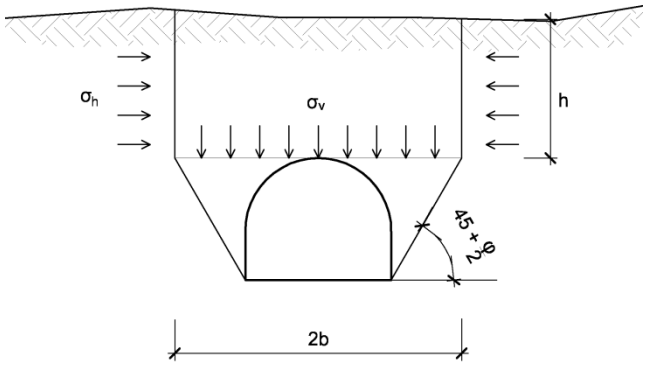

Figure 1. Terzaghi's assumption to estimate the vertical load on the tunnel support.

An alternative solution for the same problem is represented by the ground reaction curve illustrated in Figure 2, where the vertical settlement on top of the tunnel is given as a function of the radial supporting pressure.

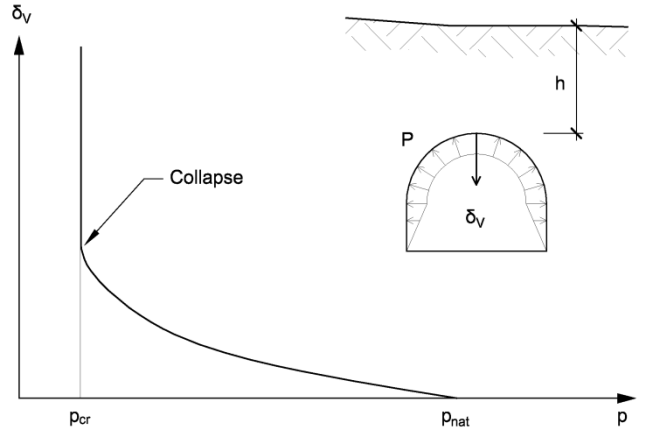

Figure 2. Critical pressure p_{cr} estimated with a ground reaction curve.

The model used for calculating the ground reaction curve presented in Figure 2 considers only a radial pressure on the roof and in the upper part of the vertical side walls, since at the floor the support cannot carry a significant load due to the missing curvature.

With the ground reaction curve of Figure 2, which includes the gravity, a non-circular tunnel, the presence of the surface and a non-uniform distribution of the radial pressure, it is possible to estimate the critical pressure p_{cr} to guarantee the tunnel stability. For dimensioning the support and subsequently the lining, it is obviously necessary to consider an higher pressure than p_{cr} in order to assure adequate safety margins.

The advantage of this type of analysis technique is that the safety margin of the support can be clearly identified. If the analyses of the tunnel equilibrium were limited by assuming a stress reduction factor or by allowing a certain amount of displacement, the ratio between the actual loading and the critical pressure, p_{cr} would remain unknown.

The comparison between both models shows that the vertical stress given by Equation 1 is higher than the critical pressure of the ground reaction curve. Calculations have been performed for different values of friction angle, cohesion, overburden (ranging from 0.5 to 1.5 the tunnel span) and of the initial stress state (ratio between horizontal and vertical stress in the range of 0.5 to 1.0). Similar results are obtained only if the coefficient of lateral pressure, λ in Equation 1 is clearly larger than one, whereas Terzaghi (1943) suggested to consider a value around 1, according to the trap door experiment results. The reason of said difference can partially be explained with the failure shape assumed by Terzaghi (Fig. 1), which produces a slightly larger silo than the one obtained in numerical models; furthermore, it might be due to the fact that the displacements around the tunnel are directed towards the excavation while this phenomenon is not simulated in the trap door experiment. The coefficient of lateral pressure, λ represents the ratio of the horizontal to the vertical stress, where latter is inside the silo and already reduced compared to the undisturbed initial vertical stress. Theoretically, the parameter λ might correspond to Rankine's passive ratio. For the value of lateral earth pressure, there are still existing uncertainties (Tien 1996).

2.2 *Effect of in situ stress anisotropy*

A second application example showing the usefulness of the ground reaction curves is represented by a circular tunnel excavated in a ground with anisotropic initial stress conditions. Figure 3 shows the ground reaction curves for the vertical and the horizontal displacements, respectively, of two points located on top and at the side of the tunnel. For sake of simplicity the effect of gravity was neglected.

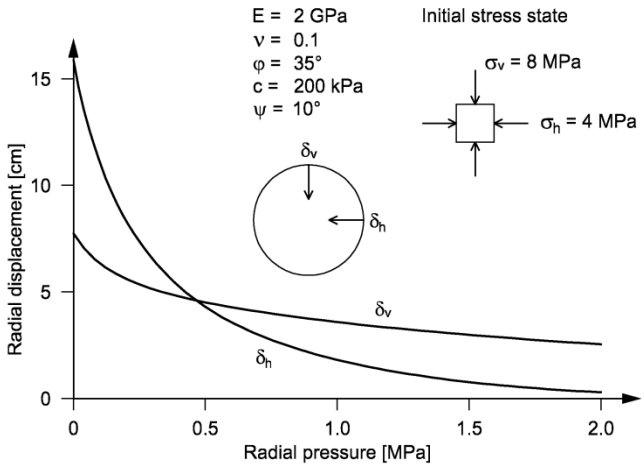

Figure 3. Ground reaction curves in case of stress anisotropy

The curves of Figure 3 are derived considering an anisotropic radial pressure within the tunnel during stress release, where the ratio between the horizontal and the vertical pressure remains the same like the initial ratio

involving the in situ stresses. The progressive stress release was thus performed with a uniform deconfining ratio.

It is interesting to observe that for high values of radial pressure the vertical wall displacement is larger than the horizontal one, while the contrary occurs for lower pressures. This differential behavior is due to the opposed contribution of elastic and plastic deformations in this particular case. The elastic component leads to larger displacements in direction of the highest stress release, i.e. in the vertical one. The plastic component, at contrary, produces more displacements horizontally, while the deviation of the vertical stress gives rise to high tangential stresses producing a more extensive failure zone.

The deconfining ratio is widely used to simulate in 2D-analysis the convergence before support installation. With this technique, however, the horizontal pressure remains lower than the vertical one even if the horizontal displacements are clearly larger than in the other direction. This result is against the basic principles of the behavior of underground excavations, where the load of the ground depends primarily on the magnitude of displacement, which is avoided by the support.

In the example of Figure 3 a radial pressure not greater than 200-300 kPa seems to be realistic for the tunnel equilibrium. For many cases, larger displacements in direction of the lower in situ stress have thus to be expected. In the same direction also a greater pressure on the support is expected, provided that the zone of the tunnel face and the statically indeterminate interaction between the support and the surrounding ground is not able to compensate the effect of the initial stress anisotropy.

This simple example shows the risk of using the deconfining ratio for tunnel analyses. This conclusion is generally valid. For example, similar considerations as the ones discussed here can be exposed when a horseshoe tunnel is analyzed: typically a high vertical load is obtained on the floor. This load will lead to an over-dimensioning of the bottom slab or the invert.

2.3 *The ground reaction curve at the face*

Amberg & Lombardi (1974) developed a method to analyze the behavior at the tunnel face in a 2D model. By this, the analysis technique given by ground reaction curves was considerably improved. In fact the ground reaction curve at the face allows to better estimate the displacement before installation of the support, which is an essential aspect for estimating the support pressure in case of expected displacements. The method represents an effective alternative to more complex 3D-analyses.

The model considers the longitudinal stresses, which are deviated by the presence of the tunnel. The curvature of the longitudinal stress trajectories leads to deviation forces within the rock mass around the cavity, which at the face are directed outward. Figure 4 shows the basic principle of the model.

The shape of the longitudinal stress trajectories, and thus the resulting deviation forces (massic forces), depend on the extension of the plastic zone around the tunnel. This is due to the fact that within the plastic zone the longitudinal stress is reduced by the rock strength (same as the tangential stress).

The presence of additional stabilizing forces within the rock mass allows to obtain a more favorable equilibrium than those for the simple 2D-case behind the face. The ground reaction curve obtained considering said deviation forces indicate less displacements for the same support reaction.

The model proposed by Amberg & Lombardi suggests additionally the way to consider the support pressure at the face. In fact, it can be assumed that the contribution of the support measures, e.g. rock bolts, steel ribs or shotcrete layers, is practically negligible at the face. However, a certain radial pressure is provided by the strength of the unexcavated core of rock, just ahead of the face. Since the ground reaction curve at the face is calculated for a 1 m thick trench, the 0.5 m thick not excavated rock portion is called half core. Its strength corresponds to the half of the compressive rock mass strength.

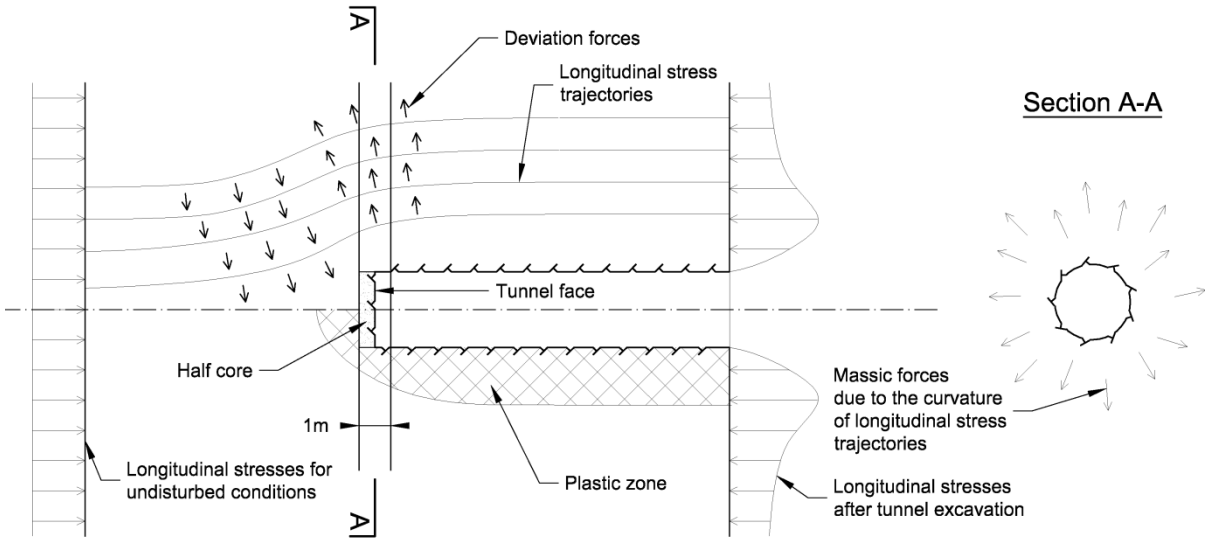

Figure 4. Equilibrium at the tunnel face.

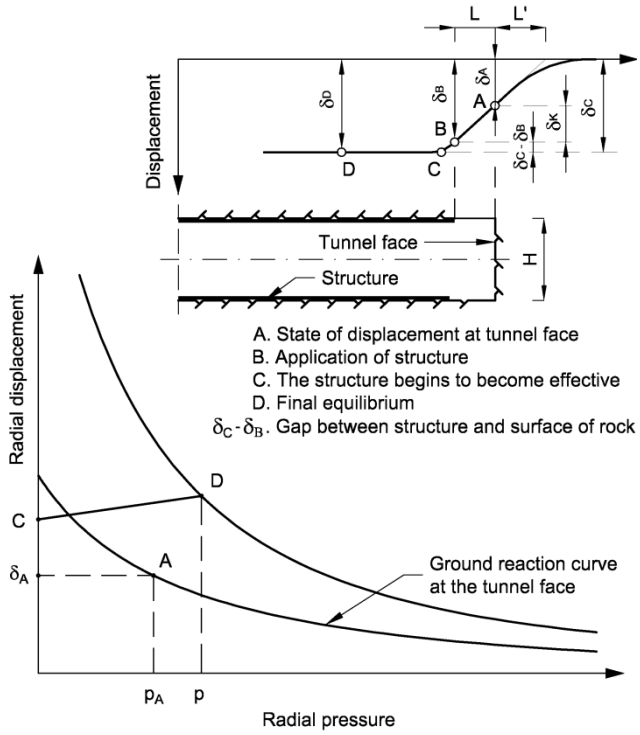

Figure 5. Ground reaction curve at the face to better estimate the final support pressure taking into account the construction procedure (p_A being the half core strength).

The advantage of this model is that the displacements at the face are calculated as a function of the in situ state of stress and the rock mass properties. The results match very well the results obtained by 3D analyses.

The purpose of the ground reaction curve at the face is to assess the most probable pressure acting on the support, taking the construction procedure as shown in Figure 5 into account. In this respect, the method presents a certain similarity with the convergence-confinement method proposed by the French school (AFTES 2002).

3 GROUND REACTION CURVE AS EQUILIBRIUM BOUNDARY

3.1 *Basic principle*

As shown in Section 2 the ground reaction curve is a very practical and useful analysis technique that supports the estimation of the equilibrium after tunnel excavation. However, it also involves some limitations and is thus not applicable to all analysis cases. In fact the ground reaction curve represents the lower boundary of the possible tunnel equilibria. Consequently, in order to avoid possible underestimations of the load acting on the support, the following basic principle should be considered: the ground reaction curve is generally determinative in the case, for which the stresses in the plastic zone around the cavity, i.e. where the rock mass strength has been exceeded, are at the failure limit. However, this condition is only valid for a monotone reduction of the support pressure. In case of a subsequent increase of the radial pressure the rock mass around the cavity is re-compressed and the stress state diverges from the failure limit. Figure 6 shows this basic principle.

The mentioned re-compression of the ground might occur for several reasons, especially if a rigid lining is placed. The most evident cases are swelling and creep. Other possible conditions might be contact grouting, in particular if an annular gap between lining and rock mass can be grouted in one step, excavation in a low permeable and soft ground or placement of a rigid lining near the tunnel face in case of squeezing ground. In fact, also the loosening up of rock beyond the roof may lead to an equilibrium above the ground reaction curve.

Finally also external reasons may cause an increasing of the support pressure as the collapse of an adjacent tunnel section or the excavation of a new tunnel near an existing one.

3.2 *Effect of rock swelling*

A frequent situation that can cause a re-compression in radial direction is the case of rock swelling. Figure 7 shows two possible equilibria: one obtained with a deformable support, the second with a rigid lining.

In case of a deformable support, swelling produces a displacement without any changes in stress. Thus, after rock swelling the stresses remain at failure limit and the final equilibrium (point B in Figure 7) lies on the ground reaction curve.

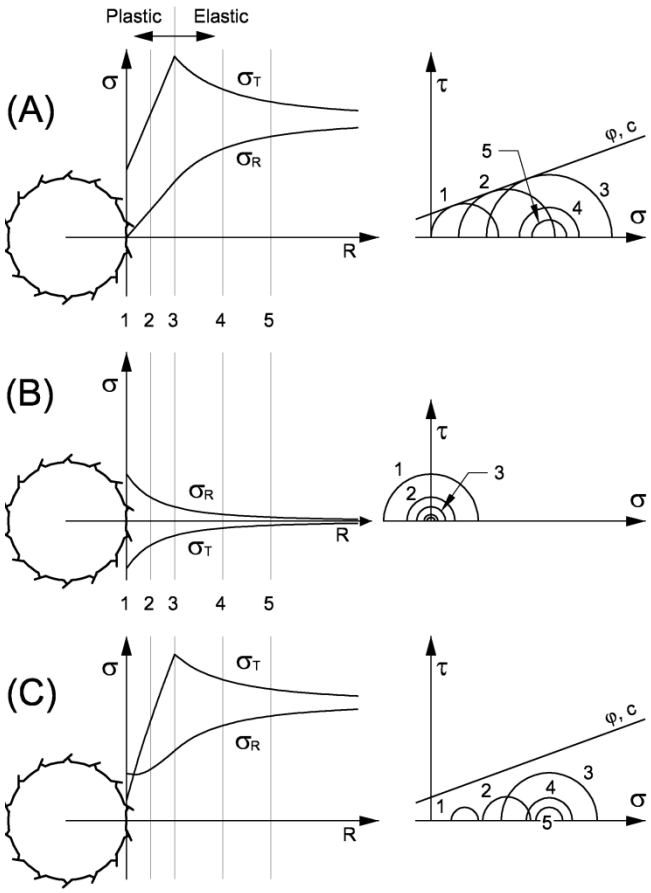

Figure 6. Stress state after excavation (A), effect of an internal radial pressure (B), final equilibrium after excavation and subsequent increasing of the radial pressure (C).

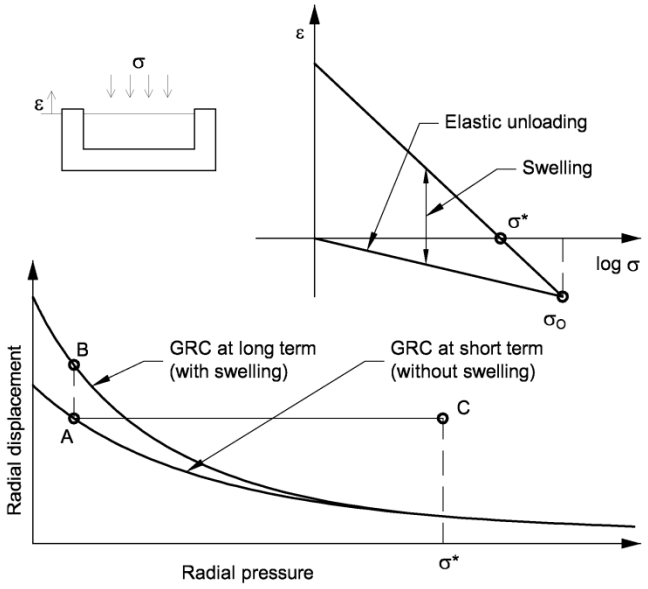

Figure 7. Equilibrium of a tunnel in case of rock swelling: A equilibrium at short term, B equilibrium in case of deformable support (e.g. pre-stressed anchors) and C in case of rigid lining (swelling pressure σ* determined according to Huder-Amberg's swelling test).

In case of rigid, thus resistive lining placed before swelling occurs, there is no volume increase and consequently the full swelling pressure can practically develop. The final point C clearly lies above the ground reaction curve. The stress state is not anymore at failure limit, since the swelling caused stress increase in all directions.

The equilibrium point C cannot be obtained if referring solely to the theory of the ground reaction curve, but also the real stress path must be considered.

3.3 *Usual tunnel construction sequence*

The re-compression of the ground and the associated moving of the equilibrium point from the ground reaction curve is a common effect, since it takes place practically for any tunnel construction. Initially, this statement might sound peculiar, since it puts the classical theories aimed to define the equilibrium in tunneling somehow into question.

In every tunnel the radial pressure at excavation radius is in fact never only always decreasing. From the initial stress ahead of the face the radial stress decreases to the minimum value in the excavation zone just behind the face. Subsequently, an increase of radial stress occurs while support measures are put in place. Any additional support measure, if placed at a distance from the face where convergences still occurs, produces a certain radial pressure increase causing a re-compression of the surrounding ground. In case of squeezing ground this phenomenon might lead to a very sensitive rising of the radial pressure acting on the lining, as shown in Figure 8.

The results of Figure 8 have been obtained by means of a 3D-simulation of the tunnel advance considering a simple elasto-plastic model based on the Coulomb yield criterion. The final equilibrium (Point A) is thus not influenced by any other possible effects like creep, consolidation or swelling.

According to the model presented in Section 2.3, the effect of re-compression observed in 3D-simulations can be explained also in using a 2D-model: in this case the re-compression is caused by the progressive reduction of the outward directed massic forces, resulting from the deviation of the longitudinal stress, with increasing distance from the tunnel face. As consequence of this re-compression, which actively originates in the rock, the support pressure on the lining increases.

The effect of the stress path on the final equilibrium of an underground excavation was, probably for the first time shown by Amberg (1999). Subsequently, Cantieni & Anagnostou (2009) presented a detailed and extensive analysis of this behavior.

The re-compression of the rock mass discussed above might also explain the phenomenon which was erroneously interpreted as a "climbing back up" of the ground reaction curves. In fact the ground reaction curves, providing the stress state remains at failure limit, never shows a radial pressure increase with increasing displacements, neither with dilatancy nor with a progressive reduction of rock properties. This is also because the radial pressure is considered as the independent variable.

The phenomenon of the re-compression is mostly evident in case of squeezing ground, in particular if a relatively rigid support is installed while the convergences are still ongoing. Compared to the ground reaction curve of Figure 8, the 3D-analysis indicates a 2.5 greater support pressure (from 0.95 to 2.4 MPa). This difference is absolutely not trivial.

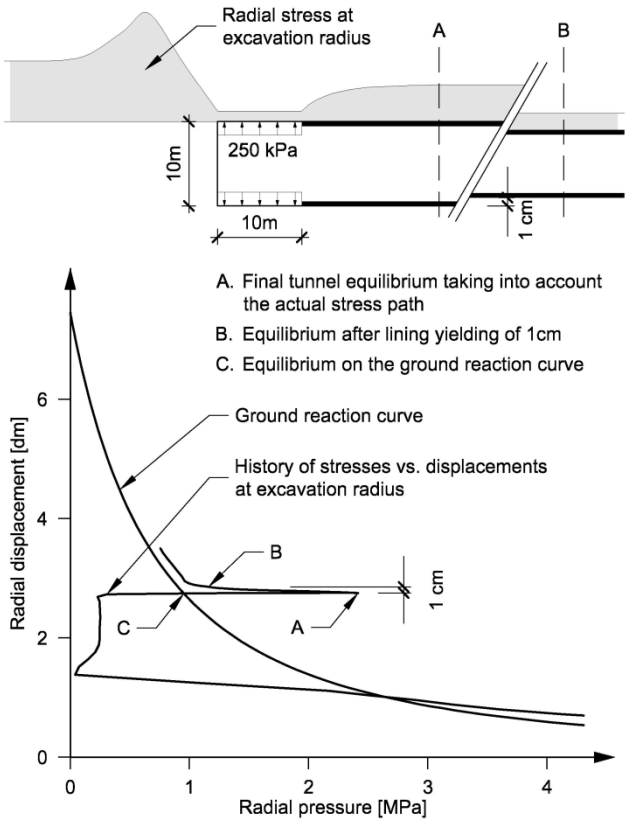

Figure 8. Ground reaction curve and final equilibrium taking into account the actual stress path during tunnel construction

Figure 8 shows the results of an additional simulation: after reaching the final equilibrium with an elastic lining (Point A), the lining softening (yielding) is simulated. After an additional radial displacements of only 1 cm, the support pressure decreases to half of the maximum pressure, i.e. from 2.4 to less than 1.2 MPa (Point B). The equilibrium point approaches again the ground reaction curve. With further yielding of the lining the stiffness of the system decreases considerably since the plasticity of the rock mass leads again to displacements.

The results exposed in Figure 8 clearly show how sensitive the phenomenon of re-compression and subsequent decompression is. In fact, the ground reaction curve can be interpreted as the curve of first decompression, which includes plastic strain, while the behavior of re-compression lies on the elastic curve of reloading, which is much more rigid.

The points along the ground reaction curve thus have to be considered differently relative to the possible equilibrium points above it. In fact, if the support would be excessively loaded to a point located above the ground reaction curve, a new equilibrium should be reached with acceptable consequences, provided the behavior of the support shows a sufficient ductility.

4 CONCLUSIONS

The dimensioning of the tunnel lining is a very difficult task mainly because of the uncertainties in basic rock mass properties, the complexity of the construction procedure and the static indetermination of the system. In underground engineering, there are therefore, apart from deterministic approaches, also empirical methods widely used. They aim to reproduce the support by resemblance with existing case histories.

If relevant convergences, able to lead to the "true rock pressure", are expected, the proper analysis of the interaction between the ground and the lining becomes an essential issue. Within the available analysis techniques, the method of the ground reaction curves is the only one exploring the whole spectrum of the interaction between support pressure and displacements. This evaluation is very helpful for the optimization of the tunnel design and might in some cases avoid premature conclusions. The application of ground reaction curves is not limited to simple axial symmetric models, but it can be extended to any type of problem by the currently available computation methods. With appropriate procedures the concept also permits to identify the equilibrium point considering the excavation procedure as well as the installation of the lining.

The ground reaction curves have been successfully used for the design of innumerable underground constructions. However, recent developments have shown that the actual construction procedure of a tunnel tends to lead to an equilibrium lying above the ground reaction curve. This behavior can be pointed out by simulating the tunnel advance in 3D-analyses.

It is very important to realize that ground reaction curves are derived by a progressive decreasing of the confinement pressure. Thus, every equilibrium point on the ground reaction curve corresponds to a limit, in some way similar to the active earth pressure theory, where within the plastic zone the stresses are at failure. In case of a confinement pressure increase, after stress release, an elastic re-compression of the ground can be observed. This phenomenon might be caused by rock swelling, creep or consolidation in combination with a rigid lining. Nevertheless, a re-compression might also occur in any usual tunnel construction, as soon as the very low support pressure close to the face increases due to the placing of the lining. In case of squeezing ground conditions, this effect might lead to a very significant increase of the "true rock pressure". This trend cannot be observed using simple 2D-analyses; simulations of the tunnel advance in three dimensions are thus required.

The relevance of equilibrium points above the ground reaction curve have to be verified case by case. The present paper shows that with few additional displacements, such as a slight softening of the lining, the pressure might drop significantly so that the final point lies again close to the classical ground reaction curve. In fact, the response of the rock mass for points above the ground reaction curve behaves according to an elastic reloading curve.

This last result permits somehow to relativize the phenomenon that gives rise to significant higher support pressures than the ground reaction curve. However, for the design of underground constructions it is indicated to consider also the possibility of having equilibria above the ground reaction curve, in order to optimize the tunnel support also for the cases where this phenomenon may arise.

REFERENCES

AFTES 2002. La méthode convergence-confinement, *Tunnels et ouvrages souterrains* No 170: 79-89

Amberg, W. & Lombardi, G. 1974. An elasto-plastic analysis of the stress-strain state around an underground opening, Part II, 3rd Congress of ISRM, Denver

Amberg, W. 1999. Konzepte der Ausbruchsicherung für tiefliegende Tunnels, *Bauingenieur* Nr. 6: 278-283

Cantieni, L. & Anagnostou, G. 2009. The effects of the stress path on squeezing behaviour in tunneling, *Rock Mechanics and Rock Engineering* 42: 289-318

Fenner, R. 1938. Untersuchungen zur Erkenntnis des Gebirgsdruckes, Glückauf, 74

Lombardi, G. 1966. Contributions at the 1st Congress of ISRM, Volume III - Events and discussions, Lisbon

Tien, H.-J. 1996. A literature study of the arching effect, Thesis at the Massachusetts Institute of Technology

Terzaghi, K. 1943. Theoretical soil mechanics, *John Wiley and Sons*, New York: 66-76