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LE RESSAUT NOYÉ DANS UN ÉLARGISSEMENT BRUSQUE

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The investigation presented by Prof. Smith is a useful contribution to the analysis of submerged jumps in abrupt expansions. The purpose of the discussion is to comment on some assumptions involved by the approach presented. Furthermore, the results of an experimental investigation conducted by the writers will be compared with the computational approach. For the notations used, reference in made to Fig. 2.

First, equation (9) contains an error in the position of parenthesis, and should correctly read

$$\left(\frac{G}{T}\right)^2 = 1 + 2F_T^2\left\{1 - \frac{1}{d/T\cdot(1-b/B)}\right\} - \frac{b}{B}\cdot\left[\left(\frac{P}{T}\right)^2 - \left(\frac{G}{T}\right)^2\right]. \tag{12}$$

Second, the author approximates Fig. 7 by the linear equation (10), although a different trend is indicated by his data. Given that the effect of P on G becomes significant for large ratios of b/B, the approach of Smith seems to be acceptable only for small ratios b/B. Based on an experimental and theoretical investigation conducted by the writers, a modified non-linear relation accounting for the width ratio b/B has been developed and will herein be compared to experiments.

The surface profile of free jumps located in prismatic channels ($b/B = 0$) has been investigated extensively such as by Rajaratnam (1962), Rajaratnam and Subramanya (1968), and Sarma and Newnham (1973). Based on these investigations, the surface profile depends exclusively on the inflowing Froude number F_1. Fig. 10 shows the experimental data of Bretz (1987).

The average surface profile for jumps in prismatic channels ($F_1 \cong 5$) could be expressed by

$$\frac{P-G}{T-G} = \operatorname{tgh}\left(2\,\frac{L_p}{L_j}\right). \tag{13}$$

Herein G is the inflowing depth of the free jump and $\operatorname{tgh}(i) = [\exp(i) - \exp(-i)]/[\exp(i) + \exp(-i)]$. Equation (13) may also be adopted for submerged jumps in non prismatic channels, as is corroborated with Fig. 7, provided $b/B \ll 1$.

The flow configuration $b/B \to 1$ occurs when somewhere downstream of the gate the channel becomes several times larger than the gate width. For the limit condition $b/B = 1$ it could be assumed that $P = T$. Thus, for any value of G one may obtain

$$\frac{P-G}{T-G} = 1. \tag{14}$$

In general $(P-G)/(T-G)$ depends on both the end position of the pier L_p/L_j, and the expansion ratio b/B. The purpose of what follows is to establish this relation by experimental means.

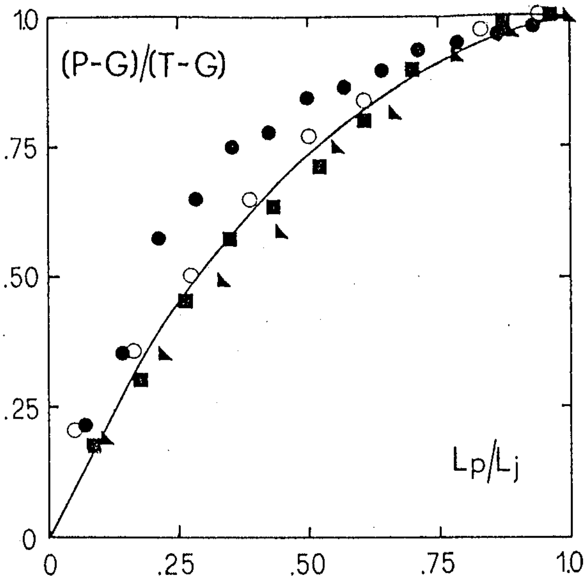

Fig. 10. Hydraulic jump, surface profile in prismatic rectangular channels according to Bretz (1987). Effect of inflowing Froude number F_1 (●)=3.92; (○)=5.23; (■)=6.36; (◣)=7.23 (——) equation (13).
Profil du ressaut hydraulique dans un canal prismatique rectangulaire selon Bretz (1987). Effet du nombre de Froude amont.

The experiments were conducted in a rectangular PVC test flume with an expansion ratio of $b/B = 0.5$. A 0.5 m wide sluice gate was located 0.8 m upstream from the channel expansion. The water depths obtained at the pier tail are shown in Fig. 11(a). The experimental facility did not allow the investigation of the domain $L_p/L_j < 0.2$. However, it is seen that the remaining data define a curve which seems to have its origin at $(P-G)/(T-G) = 0.5$ for $b/B = 0.5$. Based on these experiments and on the previously discussed limit conditions, the flow depth at the downstream end of the pier could be approximated as

$$\frac{P-G}{T-G} = \frac{b}{B} + \left(1 - \frac{b}{B}\right) \cdot \mathrm{tgh}\left(2\frac{L_p}{L_j}\right). \tag{15}$$

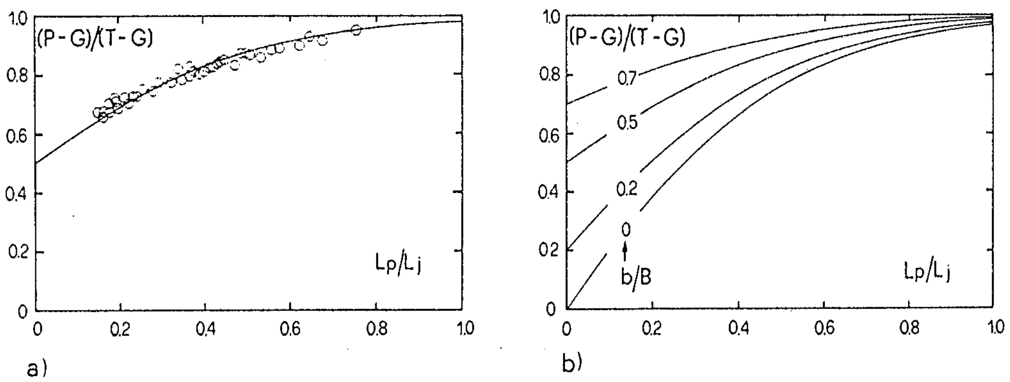

Fig. 11. Flow depth at the pier tail. a) (○) writers' experimental data for $b/B = 0.5$, (——) equation (15); b) equation (15) for different values of b/B.
Hauteur d'eau à laval immédiat de la pile. a) (○) données expérimentales des auteurs pour $b/B = 0.5$ (——) equation (15); b) equation (15) pour différents rapports b/B.

Equation (16) is plotted in Fig. 11(b) for different values of b/B including $b/B = 0$ (equation (13) and $b/B = 1$ (equation (14).

In order to predict the flow depths G and P for given values of F_T, b/B and L_p/L_j, equation (15) may be expressed as

$$\frac{P}{T} = k_1\left(1 - \frac{G}{T}\right) + \frac{G}{T} \tag{16}$$

with k_1 as the expression on the right hand side of equation (15) or equation (10), depending on the surface profile considered. A second parameter

$$k_2 = 1 + 2F_T^2\left\{1 - \frac{1}{d/T(1 - b/B)}\right\} \tag{17}$$

according to equation (12) involves only design quantities. Equation (12) becomes therefore

$$\left(\frac{G}{T}\right)^2 = k_2 - \frac{b}{B}\left[\left(\frac{P}{T}\right)^2 - \left(\frac{G}{T}\right)^2\right]. \tag{18}$$

Introducing equation (16) into equation (18) yields

$$\left(\frac{G}{T}\right)^2 \cdot \left[2 - k_1 - \frac{B}{bk_1}\right] + \frac{G}{T}[2k_1 - 2] + \frac{k_2}{k_1}\cdot\frac{B}{b} - k_1 = 0 \tag{19}$$

which could be solved for G/T.

In Fig. 12, the results obtained from equation (19) and equation (10) are compared to the writers' experimental data. Therein, index "p" refers to predicted, and index "x" to experimental values. As regards the flow depth P at the end of pier, Fig. 12(a) shows an improvement of the present prediction. However, the prediction of G is similar in both the author's, and the writers' approaches, as the effect of the non-linear surface profile on equation (12) is only moderate. Further reaching computations indicate that the author's approach for G is suited for $b/B < 0.3$. For larger width ratios b/B, and for the computation of P, the present approach may be recommended, however.

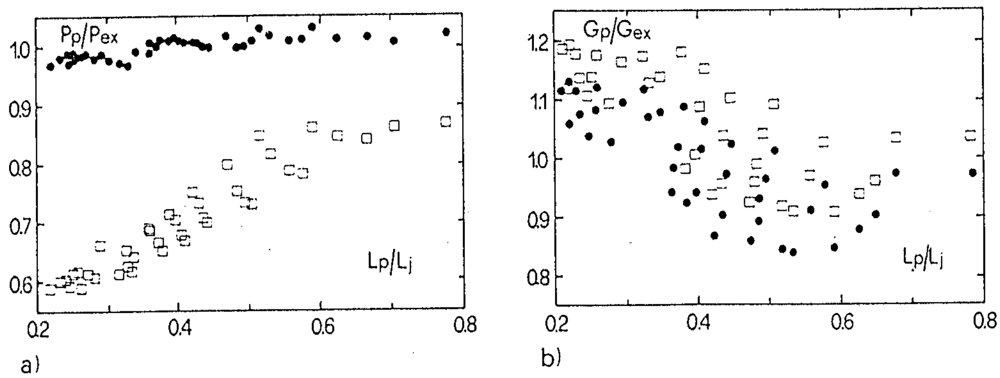

Fig. 12. Ratio of predicted and experimental values as a function of relative position L_p/L_j for $b/B = 0.5$. a) Flow depth P at the pier tail, b) flow depth G at the gate according to (□) equation (10) and (●) equation (15).

Rapport des valeurs calculées et expérimentales comme fonction de la position relative L_p/L_j pour $b/B = 0{,}5$. a) Hauteur d'eau P et b) hauteur d'eau G selon (□) l'équation (10) et (●) l'équation (15).

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Reply by the author

The writer wishes to thank Bremen and Hager for their thorough discussion and the very appropriate additional data presented on the subject. This has added considerably to the value of the study.

The work by Bremen and Hager includes results for a larger b/B ratio, considerably extending the tested range for this variable. The writer's work was intended for applications where the abrupt expansion occurs at the end of a gate supporting pier. In this case, b/B rarely exceeds 0.2. Larger b/B ratios could occur if the flow from a single gate was discharged into a much wider channel, provided the parallel sidewalls at the gate section were terminated downstream at a length shorter than the roller. If the jump is completed before the abrupt expansion begins, the case simply reverts to $b/B = 0$, for which

$$\frac{G}{T} = \sqrt{1 + 2F_T^2\left(1 - \frac{1}{d/T}\right)} \tag{20}$$

This is the equation for a submerged hydraulic jump between parallel sidewalls (Chow, 1959). As noted by Bremen and Hager, the closing parenthesis on equation (9) was misplaced. It should also be corrected in the example at the center of page 266. The theory, results and numerical example in the paper all agree with the correct equation (shown as equation (12) by Bremen and Hager). Also, strictly speaking, the sign should be reversed on either the left side or right side of equation (3).

Bremen and Hager have noted, quite correctly, that the actual jump profile is non-linear and that it varies somewhat with the Froude number. Equation (10) was not intended to suggest otherwise, but rather that the value of P using this equation gave a better design value for G if the pier tail force F_p in equation (3) was calculated using the equation

$$F_p = \frac{b\gamma P^2}{2} \tag{21}$$

Although the P value in the writer's equation (10) is less than the measured value, this was found necessary in order to compensate, in equation (4), for the fact that friction was neglected and that the pressure on the pier tail is less than hydrostatic. In addition, there is a small safety margin included, in that it is important that the water depth at the gate G not be underestimated, since this would give an apparent head h, in Fig. 1, greater than would actually be available. Any overestimate of F_p will lead to an underestimate of G.

As shown by the discusser's Fig. 12, however, equation (10) would be inaccurate and too conservative for large b/B ratios. When $b/B \to 1$ and $F_T \to 0$, the water downstream from L_p will become quiet, level transversely, and the pressure distribution on the pier tail will become more closely hydrostatically distributed. In the limit, P will equal T, independant of L_p, as given by equation (14).

The writer agrees that the discusser's equation (15), shown plotted in Fig. 11(b), and which is strongly supported by the data for $b/B = 0.5$ shown in Fig. 11(a), probably gives a better value for P for a wide range of b/B values. However, whether this is the value of P that should be used in equation (4) for purposes of calculating G is open to argument. For the reasons quoted previously, the writer would be inclined to use

$$\frac{P-G}{T-G} = \frac{L_p}{L_j} + \frac{b}{B}\left(\frac{L_j - L_p}{L_j}\right) \tag{21}$$

This equation becomes equation (10) when $b/B \to 0$ and becomes equation (14) when $b/B \to 1$, thus avoiding the limitation that equation (10) is valid for small values of b/B only.

The writer reviewed his 15 plots of $(P-G)/(T-G)$ vs L_p/L_j and Froude number for $b/B = 0.1$, 0.2 and 0.3, and found that the best agreement with the discusser's Fig. 11(b) occurred for high values of F_T (greater than 0.18). Values of $(P-G)/(T-G)$ for smaller Froude numbers generally fell below the curves on Fig. 11(b). This could be considered as a further justification for using equation (21). However, as pointed out by Bremen and Hager, the effect of the surface profile shape is less important for small values of b/B. For example, with $b/B = 0.1$, the calculated value of G would be the same whether $(P-G)/(T-G)$ was calculated from equation (10), (14) or (21).