"PRACTICE AND THEORY OF ARCH DAMS"

Proceedings of the International Symposium on Arch Dams
Nanjing (China), October 17-20, 1992

ON THE LOMBARDI "SLENDERNESS COEFFICIENT" FOR ASSESSING THE CRACKING POTENTIAL OF ARCH DAMS

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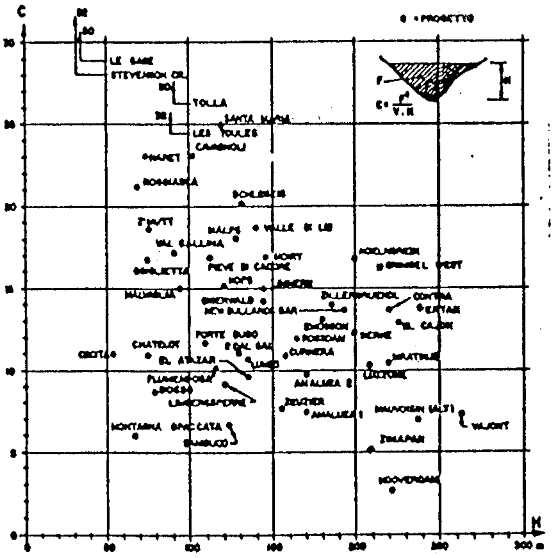
In references [1] [2] Lombardi proposes the "slenderness coefficient":

$$C = \frac{F^2}{V\ H}\ \text{, see Fig. 1,}$$

to be used, in conjunction with height H, to judge the susceptibility of an arch dam to cracking, with particular emphasis on shear effects.



Here we want to present heuristic justification for the limits statistically found in the plane (C,H) for dams without troubles (see refs. above).

Consider a dam defined as in Fig. 2. (transverse section of the valley).

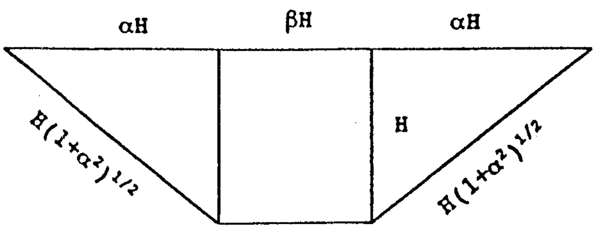


Fig. 2

Average shear stress along the foundation surface A ($A = 2hH\left(\sqrt{1+\alpha^2}+\frac{\beta}{2}\right)$, with h = average thickness) is evaluated neglecting the arch action: in fact, consideration of the arch effect would reduce the shear contribution, but since this is certainly not uniform in intensity and direction, one should increase the average to evaluate the local maximum shear. Assuming the two effects to roughly compensate each other, we obtain an effective shear stress of the order of:

$$(1) \qquad \tau \approx 1.25\,\frac{\gamma_a\left(\frac{H}{2}\times\beta\cdot H\cdot H+\frac{H}{3}\cdot\alpha H\cdot H\right)}{2hH\left(\frac{\beta}{2}+\sqrt{1+\alpha^2}\right)}$$

$$= \frac{1.25\ \gamma_a}{2}\,\frac{\frac{\beta}{2}+\frac{\alpha}{3}}{\left(\frac{\beta}{2}+\sqrt{1+\alpha^2}\right)(\alpha+\beta)}\ CH\ ^{(*)}$$

(taking into account that $C = \frac{F^2}{V\ H}$ and that $V = Fh$).

By taking $\tau_{max} \le 3$ MPa as an upper limit to avoid cracking it comes:

$$(2) \qquad CH \le 240\,\frac{(\alpha+\beta)\left(\beta+2\sqrt{1+\alpha^2}\right)}{\frac{\beta}{2}+\frac{\alpha}{3}}$$

(shear criterion)

In the case of Kölnbrein $\beta \approx 0.77$, $\alpha \approx 1.96$,

$$CH \le 2300.$$

With the dimensions of the dam one gets instead:

(*) The coefficient 1.25 is adopted as an average between 1.5, which would apply for a constant-section beam, and 1 which would more nearly apply in extreme cases, for inclined facings and non-uniform section.

$$CH \approx 3500,$$

definitely larger than the limit.

In the case of Valle di Lei $\beta \approx 0$, $\alpha \approx 2$;

$$CH \leq 2250.$$

With the dimensions of the dam one gets:

$$CH \approx 2500,$$

only slightly larger.

It is to be noted, however, that the crisis of the structure can take place also for tensile cracking. In this context, let's consider the arch at mid-height. For a dam only 30 m high, ($H = 30$) it would be reasonable to adopt a thickness of $h = \frac{H}{10} = 3$ m. The average compression in that arch would be:

$$(3) \quad \begin{cases} |\sigma_a| \approx \frac{1}{2} \cdot \dfrac{\gamma_a \cdot \frac{H}{2} \cdot R_a}{h} \text{ (*) and one could allow} \\ |\sigma_a| \approx 3 \text{ MPa.} \end{cases}$$

At the abutment intrados we would have $\sigma_{max} \approx 0$, $|\sigma_{min}| \approx 2\,\sigma_a$.

Now if we take a dam with the same shape, but n times larger, all stresses would increase n times. To leave the stresses roughly unchanged, one would have to increase the thickness more than linearly:

$$h \rightarrow n \cdot n^x\, h, \text{ with } x > 0.$$

A heuristic criterion could then run more or less as follows:

At the point of maximum danger for tensile cracking the stress from normal load, σ_n, should be roughly equal to the stress from bending. If this is assured in the "small" dam:

$$(4) \qquad \sigma_n \approx \sigma_m;$$

in the "large" dam it should then be, thanks to the increase in the ratio n^{1+x} in thickness:

$$(5) \qquad \frac{n\,\sigma_n}{n^x} + \frac{n\,\sigma_m}{(n^x)^2} \approx \sigma_n + \sigma_m.$$

Taking into account (4) it comes:

$$(6) \qquad \frac{n}{(n^x)^2} + \frac{n}{n^x} \approx 2.$$

The solution for x thus depends on the magnification factor, n, but it is readily found that for:

$$(7) \qquad 1 \leq n \leq 5$$

one gets:

$$(8) \qquad \tfrac{2}{3} \leq x \leq 0.734,$$

so that one is justified in taking the constant value:

$$(9) \qquad x \approx 0.7.$$

With this assumption, scaling up of dimensions (H) to a factor n entails scaling up thicknesses to a factor $n^{1.7}$ in order to maintain roughly the same stress level. Thus the scaling up of C will be to a factor:

$$(10) \qquad C \rightarrow \frac{n^4\, S^2}{n^{1.7}\, n^3\, V\, H} = n^{-0.7}\, C$$

(slenderness coefficient should be smaller for larger dams).

More specifically, from (3) and Fig. 2, under the assumption of roughly 106° angular aperture of the middle arch, so that:

$$(11) \qquad R_{co} \approx \cancel{0.8}\ 1.25\ H_0 \left(\frac{\beta}{2} + \frac{\alpha}{2}\right)$$

for the dam 30 m high, of 3 m average thickness, it comes, keeping into account (9):

$$(12) \qquad C < \cancel{1650}\ 762\ (\alpha + \beta)^{-0.7}\, H^{-0.7}$$

(normal stress criterion)

Thus, to effect a preliminary, rough evaluation of the susceptibility of a dam to cracking, one could suggest the following procedure:

a) from the shape factors α, β compute the "aspect factors" of the valley:

(*) The factor $\frac{1}{2}$ allows approximately for the fact that the arch at mid-height takes only a part of the total hydrostatic load $\frac{1}{2}\,\gamma_a\,H$.

$$(13)\quad \begin{cases} C_1 = \dfrac{(\alpha + \beta)\left(\beta + 2\sqrt{1 + \alpha^2}\right)}{\left(\frac{\beta}{2} + \frac{\alpha}{3}\right)} \times 240 \\ \\ C_2 = (\alpha + \beta)^{-0.7} \times \overset{762}{\cancel{1650}} \end{cases}$$

b) trace, on a log-log cartesian plane (abscissa log H, ordinate log C), the <u>two straight lines</u>:

$$(14)\quad \begin{cases} \log C = \log C_1 - \log H \text{ for the shear criterion,} \\ \\ \log C = \log C_2 - 0.7 \log H \text{ for the normal stress criterion} \end{cases}$$

(see Fig. 3)

The region of the log-log plane under both these two lines(*) identifies, for the valley being considered, the dams relatively safe from cracking: all points above should be regarded at risk from cracking.

It is deemed that the above considerations, while in agreement with the previous work by LOMBARDI, can further refine his criterion, by taking into account the "aspect factors" of the valley (13). Moreover, a physical basis for the statistically found subdivisions of existing dams into "safe regions" and "regions of cracking potential" in the plane (C,H) is provided.

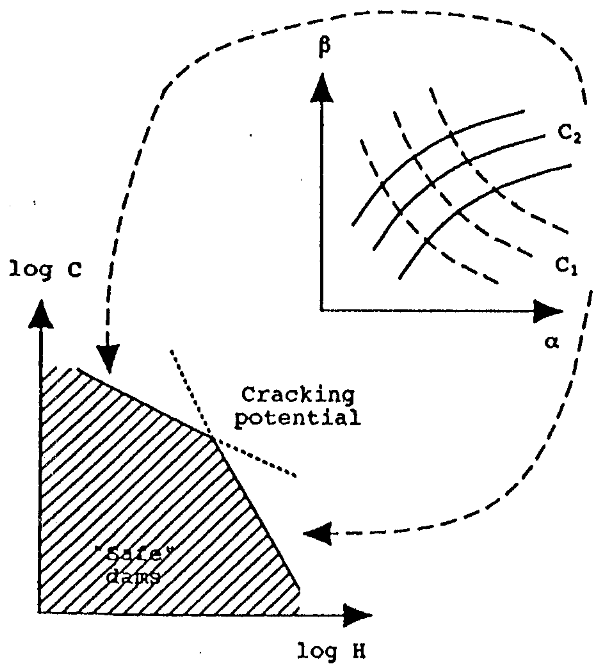


Fig. 3

(*) From Fig. 4 one can quickly pinpoint the intersection of the two straight lines in the Log C - Log H plane, so that the frontier between "safe" and "risky" designs can be immediately drawn.

Naturally, other important parameters influencing the cracking potential of arch dams are the ratio of Young moduli of rock and concrete, the angular aperture of arches, etc.

All of these should be taken together and duly considered by the designer in the light of the state of the art knowledge and of the past experience.

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[1] LOMBARDI, G., "Querkraftbedingte Schäden in Bogensperren", Wasser, Energie, Luft, No. 5/6, 1988.

[2] LOMBARDI, G.,"Kölnbrein Dam: an Unusual Solution for an Unusual Problem", Water Power & Dam Construction, June 1991.

[3] KAECH, A., LOMBARDI, G.,"Alcune considerazioni sulle dighe ad arco" (Some considerations on arch dams), Schweizerische Bauzeitung, No. 38, 1953, pp. 549-553.

[4] "Advanced Dam Engineering for Design, Construction and Rehabilitation", A Van Nostrand Reinhold Book, edited by Robert B. Jansen, 1988, pp.578-608 (Chapt. 19).

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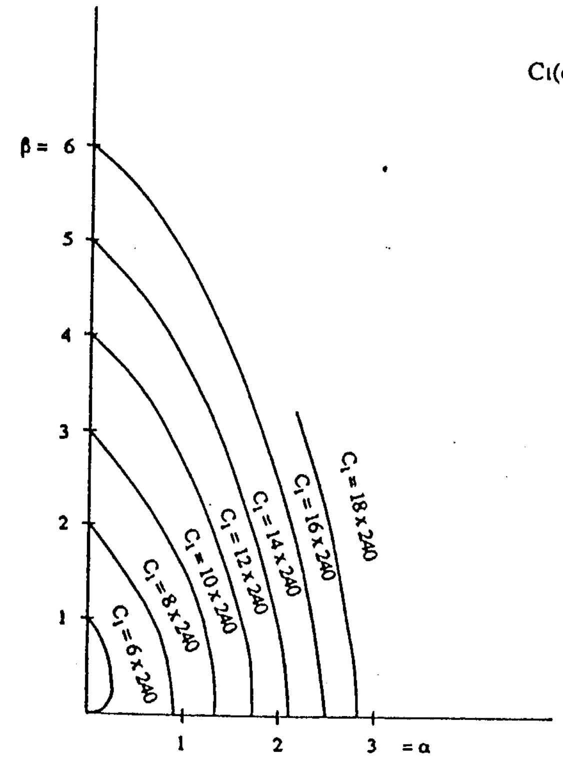

Fig. 3a

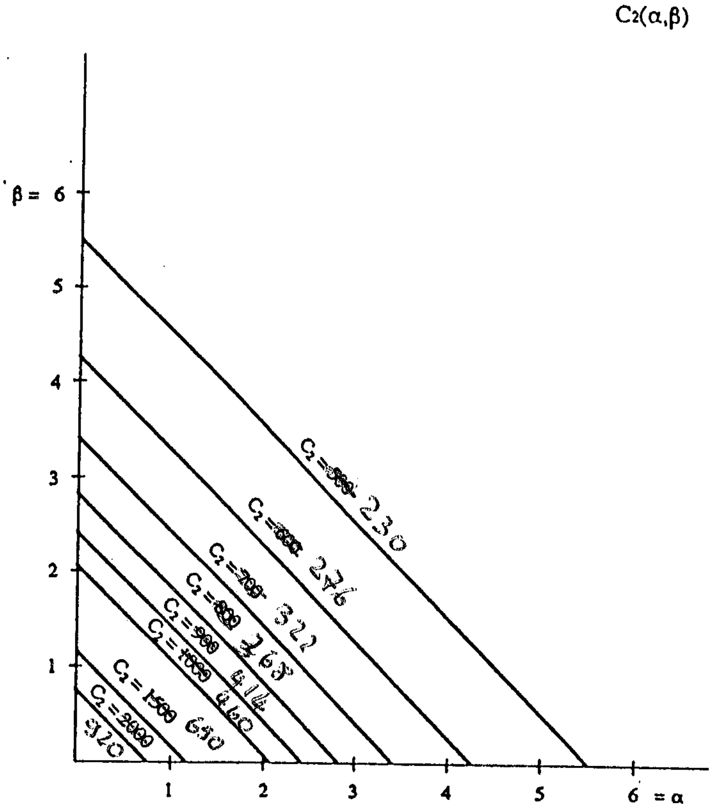

Fig. 3b

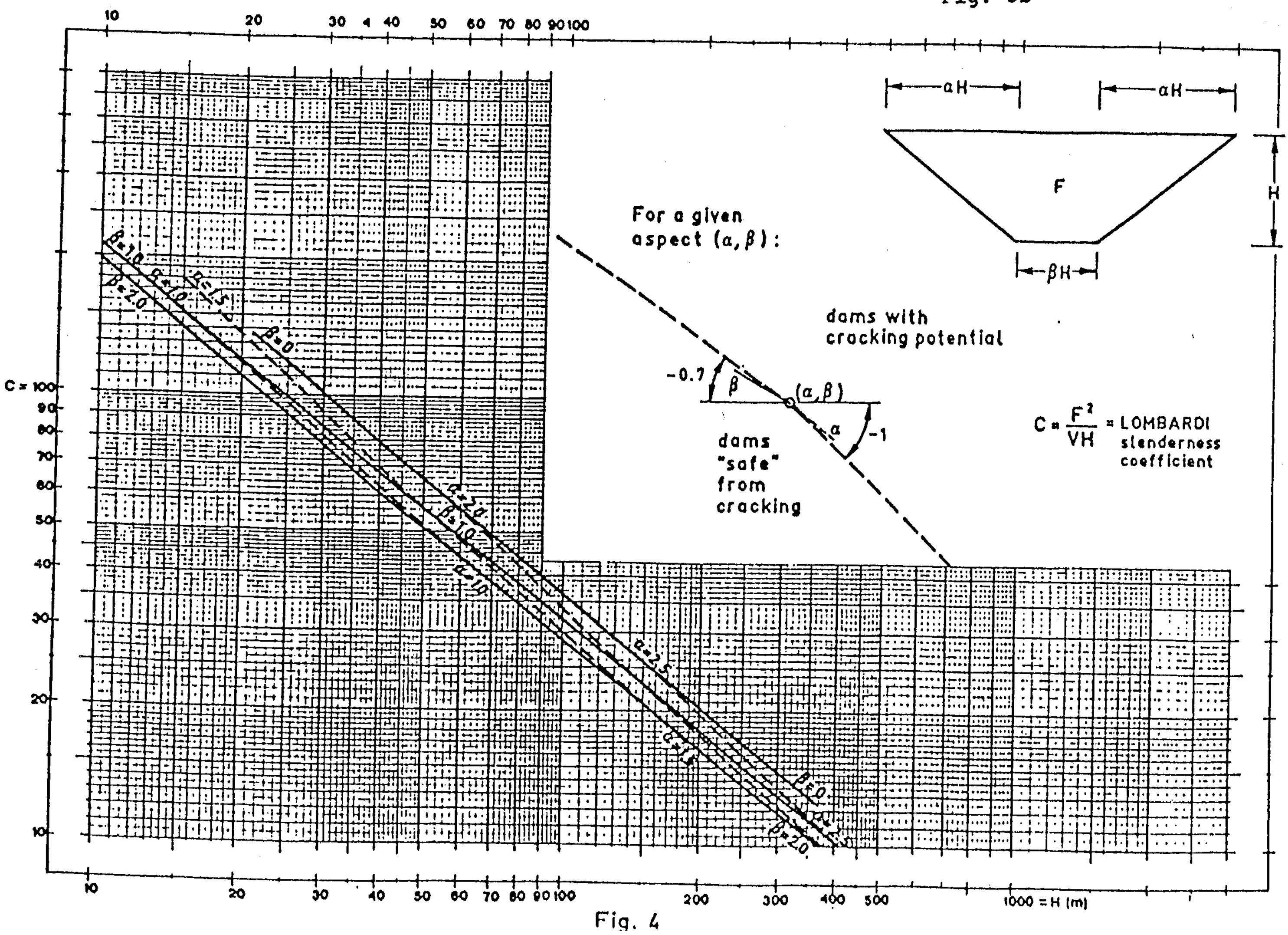

Fig. 4

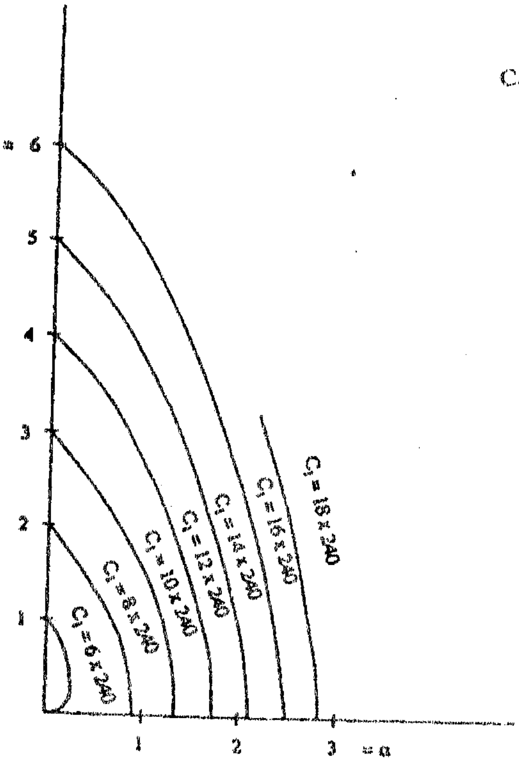

Fig. 3a

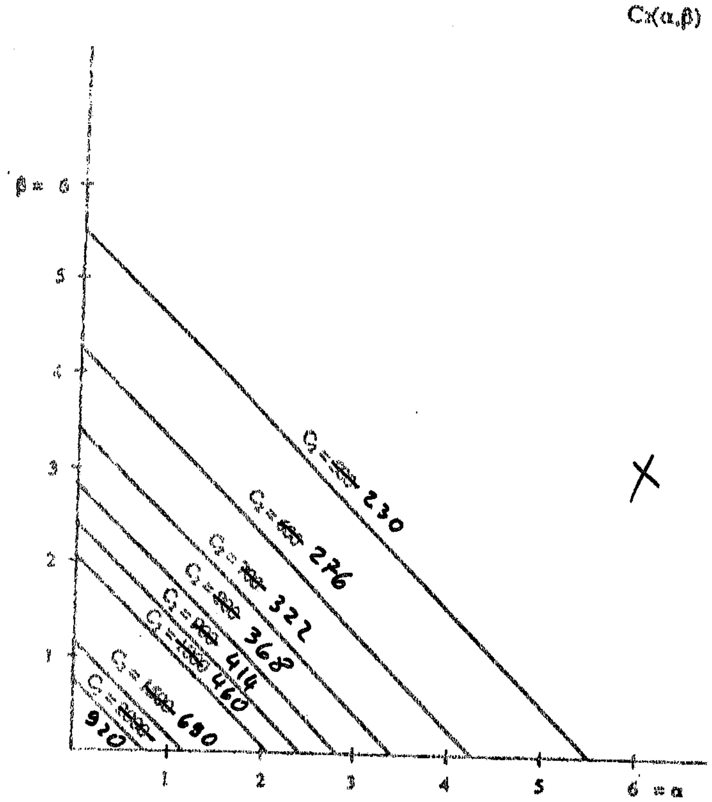

Fig. 3b

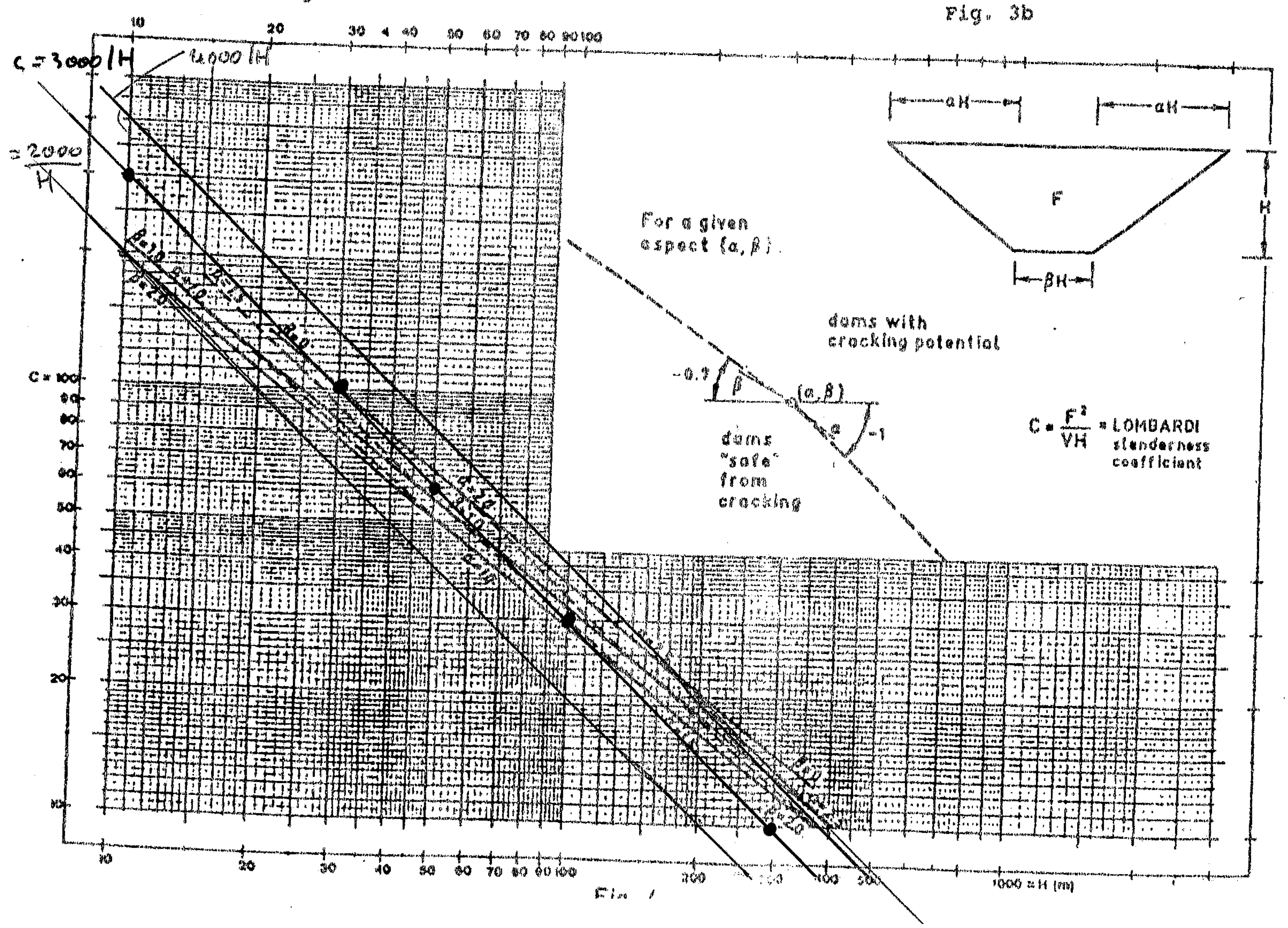

Fig. 4

ERRATA TO THE BOOK:

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**. ERRATA-CORRIGE TO THE PAPER:

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pag.	formula	ERRATA	CORRIGE
2	(11)	$R_{co} = 0.8\ Ho\ (\frac{\beta}{2} + \frac{\alpha}{2})$	$R_{co} = 1.25\ Ho\ (\frac{\beta}{2} + \frac{\alpha}{2})$
2	(12)	$C < 1650\ (\alpha + \beta)^{-0,7}\ H^{-0,7}$	$C < 762\ (\alpha + \beta)^{-0,7}\ H^{-0,7}$
3	(13)	$C_2 = (a + \beta)^{-0,7} \times 1650$	$C_2 = (\alpha + \beta)^{-0,7}\ X\ 762$
4	Fig. 3b	All the values of C_2 indicated on the figure should be reduced by a factor of $\frac{762}{1650} \cong 0.46$ (e.g. the curve labeled "500" should be labeled "230")	
4	Fig. 4	is no longer to be used.	