The FES rock mass model – Part 1

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SUMMARY

The FES model was developed during the investigations and studies carried out on the exceptional settlements of the rock foundation of Zeuzier arch dam in Switzerland. The model was originally intended to express the body law of a Fissured, Elastic, Saturated rock mass, but was successively expanded to cover a wider field of problems. The mathematical background of the model is described for the first time and some examples of computational results are presented.

Introduction

The very important and quite exceptional settlements of the rock foundation at the Zeuzier arch dam in Switzerland, produced by the lowering of the ground water table following the excavation of an exploratory adit, have been described on a number of occasions[1].

The intensive study of this case, and of the related problems, led to the staged development of the so-called FES model for rock masses. The results obtained during some applications (for example, rock grouting) has been the subject of various papers also[2]. However, the mathematical reasoning for this model as a whole has not been published before.

In the following, the essential aspects of the computations to be carried out in order to establish the model which represents the body law of a given rock mass configuration will be described. The limitations of the model will also be indicated.

1 The problem

The problem the model is intended to deal with is the provision of a mathematical tool able to simulate the behaviour of a fissured rock mass under the combined effect of changes both in the total stresses as well as in the neutral or internal interstitial water pressure. In fact, the intention is clearly to develop the body laws needed to describe as well as possible the behaviour of such a structure under said loads.

In a first step the rock matter itself is supposed to react elastically with a modulus of elasticity E and a Poisson ratio ρ. Later on some broadening of the concept was carried out to introduce the effect of a softening of the surfaces of the joints in the fissured rock mass, as well as the effect of a plasticity limit of the rock matter itself. But, in spite of these facts, the name FES for 'Fissured, Elastic, Saturated rock mass model' was kept.

[1] see bibliography on Zeuzier dam
[2] see the FES model bibliography

2 Conditions of equilibrium

Figure 1 shows two rock blocks in contact along a part of the joint surface in between. They are loaded by a total stress σ_t, while in the fissure an internal water pressure p exists. The ratio of the surface in contact with the total surface of the joint will be called 'ratio of contact or closure', α. This plays an important role as the main parameter of the rock mass behaviour during the 'serrage' or closure of the fissures under load.

Obviously, the contact 'rock to rock' stresses between the two blocks at any point need to be higher than the acting water pressure, otherwise the water would enter that portion of the joint and open it.

Incidentally, it has to be mentioned that the assumption is made that the rock matter itself does not absorb any water or, if some, so slowly that this will not affect the short term behaviour of the rock mass.

On the same figure (points '4' and '5') it may be seen how the effective stress σ_e is defined as the average value of the contact stresses which exceed the neutral pressure.

The effective stress σ_e is obviously smaller than the rock to rock contact stress σ_c, which has to be accounted for in studying the potential for sliding along the surface of discontinuity. The effective stresses and contact stresses will coincide only in the case when the water pressure is nil.

Four our purposes we will consider the average of these stresses on the entire surface. The following relationships then apply:

$$\sigma_t = \sigma_e + p$$
$$\sigma_c = \sigma_e + \alpha \cdot p$$
$$\sigma_e \leq \sigma_c \leq \sigma_t$$
$$\text{if } \alpha = 1, \; \sigma_c = \sigma_t$$
$$\text{if } p = 0, \; \sigma_e = \sigma_c = \sigma_t \tag{1}$$

3 Geometrical modelling

It appears clear that, to treat the problem mathematically, a certain number of simplifying assumptions are unavoidable. These assumptions refer to the nature and the shape of the surfaces of the two not yet loaded blocks which are in contact at a few points or, more generally speaking, to the geometry of the joints or of other discontinuities of different types.

It is obvious that any simplification of this kind will produce some distortion in the approximation of the reality. However, it is believed that the assumptions to be made, which may even be partially released later on, do not really jeopardize the proposed model.

First of all, it may be assumed (see Figure 2) that the centre line or central surface of the joint, and the opening, which is irregular, may be considered as being locally flat and that it can be approached, section by section, by a plane (Figure 2a and 2b).

Further, it is assumed that the quite irregular – but now symmetrical – surfaces of the blocks, may be fitted well enough by a mathematical expression, or at least by a few numerical values describing their shape; in fact the variable thickness of the void between the two, not yet loaded, rock blocks can best be expressed by an analytical function (Figure 2b and 2c).

Clearly, if considering a larger portion of the unloaded fissure, or surface of discontinuity, or joint, not only one but a number of contact points will exist. It should therefore be assumed that these points are distributed according to a regular (that is, hexagonal) pattern as shown in Figure 3. Obviously, such a joint does not really exist. In loaded conditions contact spots or surfaces will develop starting from said points. For the sake of mathematical simplicity, the contact spots are supposed to be hexagonal also. The distance 'a' from contact point to contact point is a characteristic element of the structure, likely to be considered as a kind of scale of the surface of discontinuity.

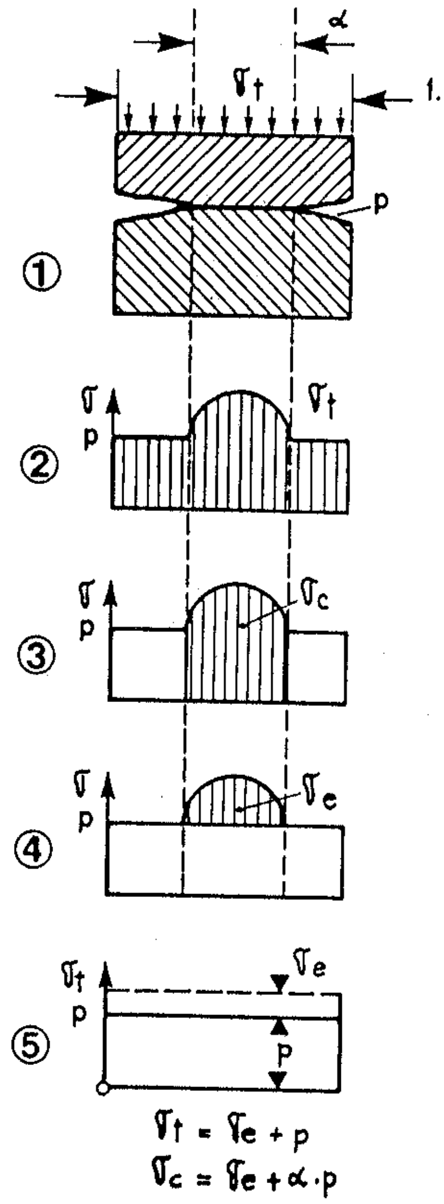


Figure 1. Basic concepts of the FES model: (1) contact of the two rock blocks under the total stress σ_t (compression) in the presence of neutral water pressure p in the joint; (2) total stress; (3) rock to rock contact stress; (4) effective stress; (5) average values for effective stress and water pressure; α ratio of closure.

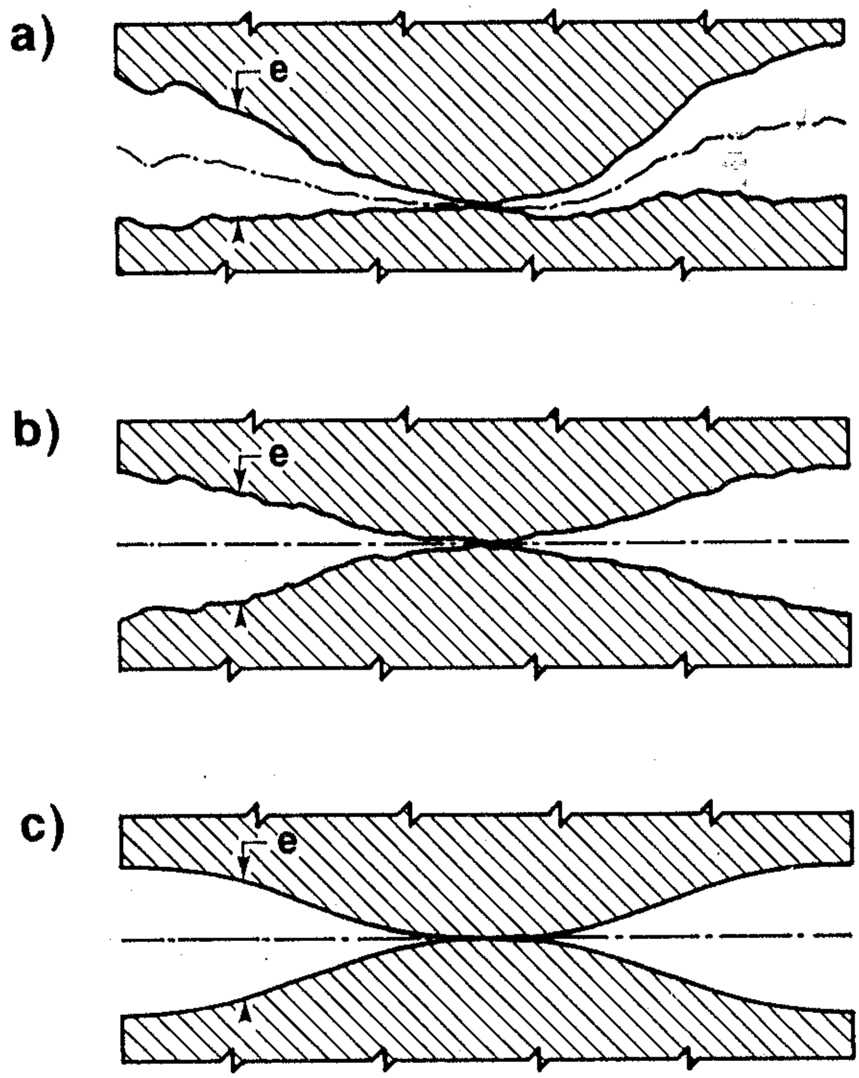


Figure 2. Approximation of the undulations of the joint surfaces: (a) actual surfaces of the joint opening with an irregular mean surface; (b) mean surface transformed to a plane, keeping the opening e; (c) best fit of the block surfaces, using an analytical formula (or a numerical vector) – equivalent smooth opening.

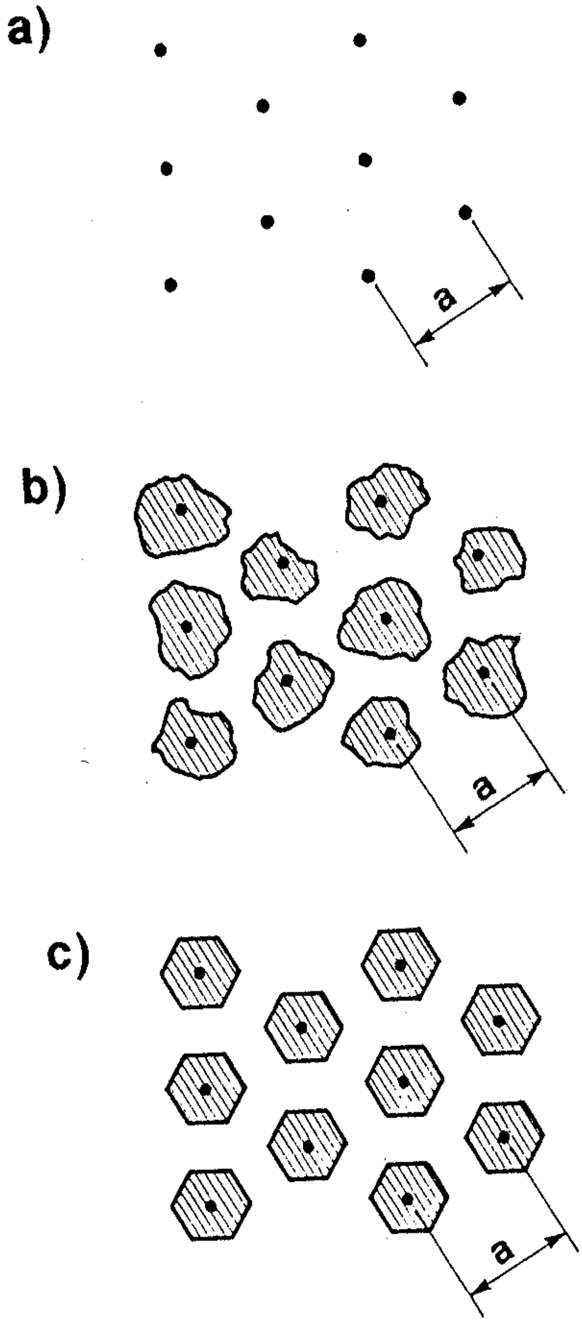


Figure 3. Approximation of the contact zone pattern: (a) geometrical pattern of the contact points (no load transmitted by the joint); (b) actual contact zones (expanding as the load on the joint increases); (c) geometrical approximation of the contact zones for a given load; a distance between contact points (scale of joint surface)

When, finally we introduce in our considerations a system of fissures parallel to each other – that means we study a series of rock layers separated by joints – we arrive at the situation defined by Figure 4. Because of the postulated symmetry, we have now to deal with a single simple hexagonal prism resting on a rigid base. On its top, a fictitious, absolutely stiff body presses downwards on the protrusion, flattening it progressively by an increasing load (Figure 4 b).

For computing purposes, the hexagonal shape of the prism may then easily be replaced by a circular one, transforming it to a cylinder with radius R (Figure 4c).

In the above described configuration, it was implicitly assumed that the undulations on both sides of the rock layer corresponded exactly. This last assumption, however, is not required, provided one is willing to introduce in the computations the additional deformations of the rock plate bent by the eccentricity of the forces acting on the upper bound in relation to the lower one. In doing this, the deformability of the fissured rock mass is slightly increased.

4 Structural analysis

4.1 Dry conditions

In a first stage of computation no water pressure will be considered. The structural analysis of the cylinder defined earlier can be carried out in two ways which in the end lead to the same result.

The first way is that of a total lateral restraint of the cylinder; this leads to the computation of the transverse forces or stresses required. The second way assumes the cylinder is completely unrestrained and computes its corresponding transverse expansion. The first case however might be preferred, as it will ease the quantification of the splitting stresses induced by the load concentration in the axis of the cylinder.

The problem as shown in Figure 5 is that of defining the distribution of the loading stresses needed to flatten the top of the cylinder to a predetermined extension and to compute the corresponding total force versus the overall compressive deformation of the cylinder, which is simply the settlement of its top.

The computational scheme uses a set of 'unit loads' of 'triangular' intensity, acting concentrically to the cylinder axis. (The various 'unit loads' do not need to have the same peak intensity.)

Figure 6 shows how a single unit load produces a deformation of the upper surface of the cylinder. Selecting a number of unit loads (for example, 20), one obtains the family of functions of influence – or deformed surfaces – reproduced by Figure 7. The computation can be done quite easily on the base of a central-symmetric finite element mesh and does not require any additional comment.

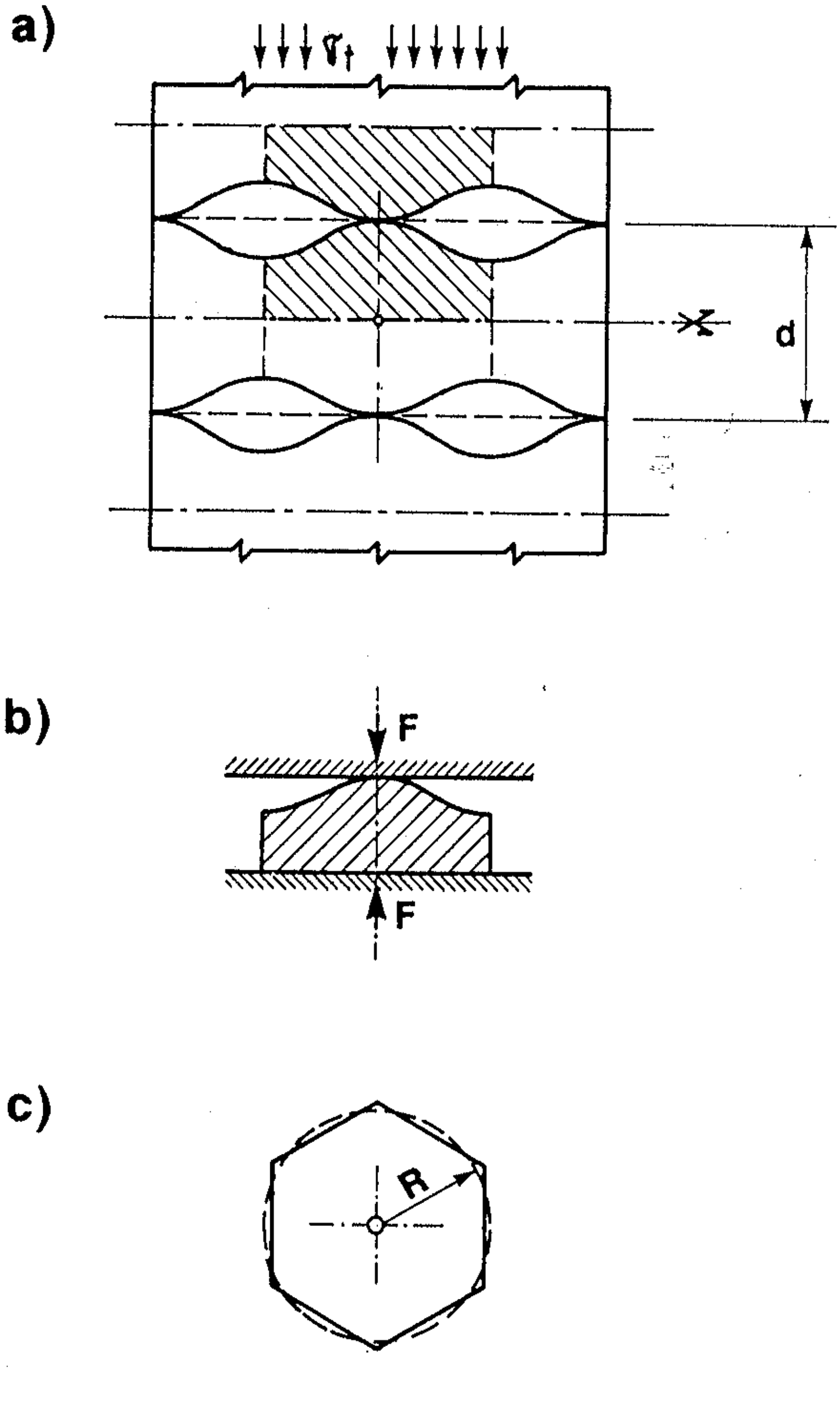


Figure 4. Defining condition for the structural computation: (a) each rock layer defined by two equal joints, with corresponding contact points; (b) a unit hexahedron resting on a stiff base, the top protrusion is flattened by the load F of a theoretical absolute stiff body; (c) hexahedron replaced by a cylinder of radius R, a unit (half) cylinder is then used in the computations; 'd' thickness of the rock layer.

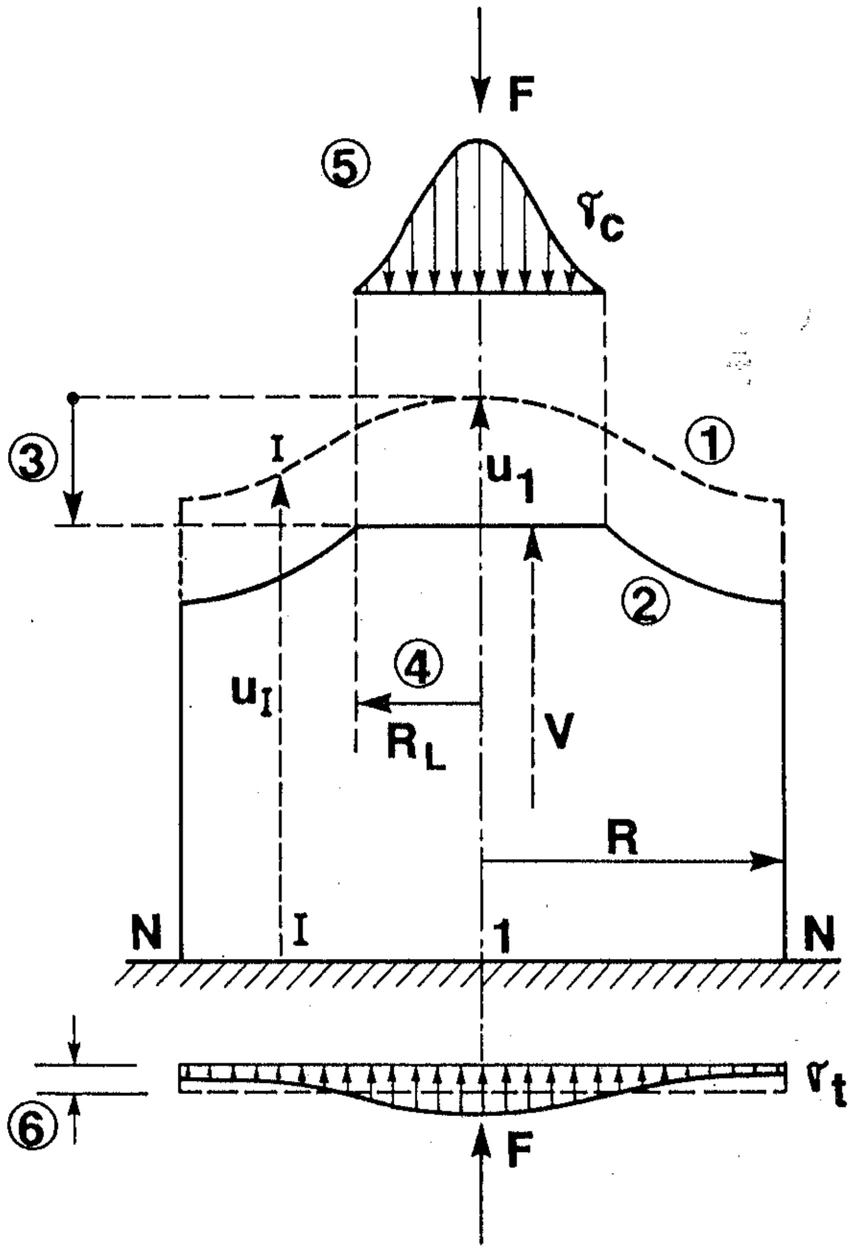


Figure 5. Flattening the protrusion on top of the cylinder: (1) initial shape of the protrusion, U_I elevation at point I; (2) deformed protrusion; (3) settlement of the top corresponding to the deformation of the fissured mass; (4) radius of the flattened zone R_L (the closing ratio $\alpha = (R_L/R)^2$); (5) the contact stress σ_c, corresponding to the total load F; (6) total stress $\sigma_t = F/(\pi \cdot R^2)$.

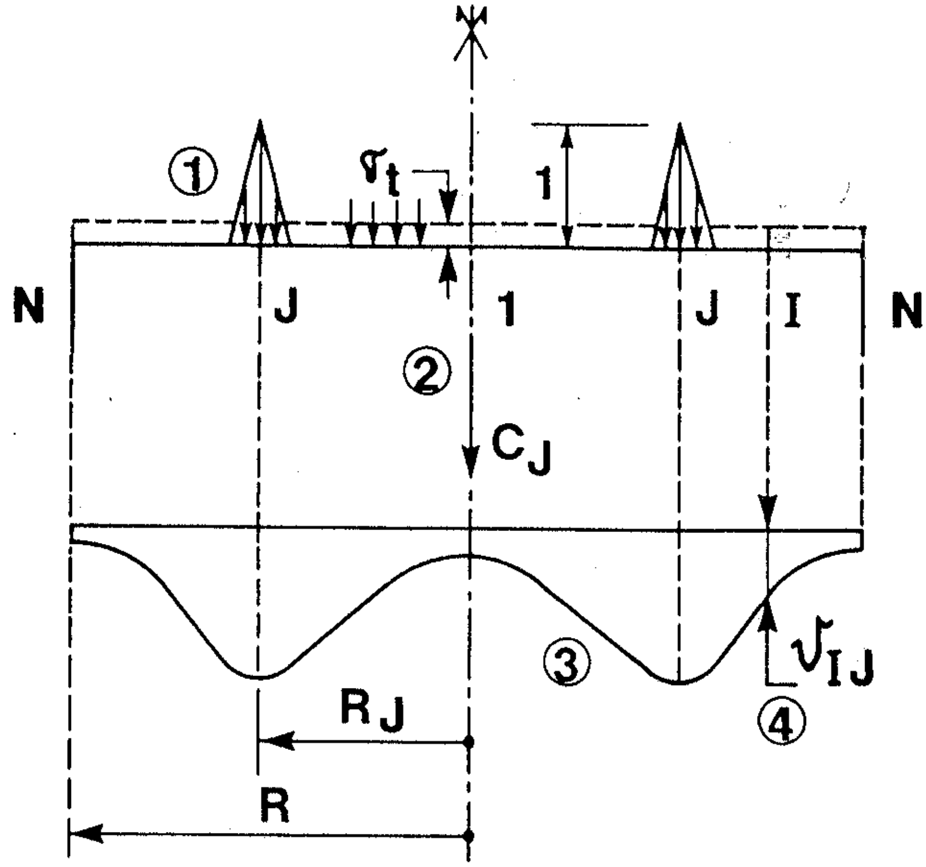


Figure 6. Deformation resulting from a typical unit load: (1) unit load J acting on top of the cylinder (mean total stress, σ_t); (2) C_J = corresponding total force; (3) deformation of the surface resulting from the unit load; (4) ϑ_{IJ} = deformation at point I resulting from unit load J.

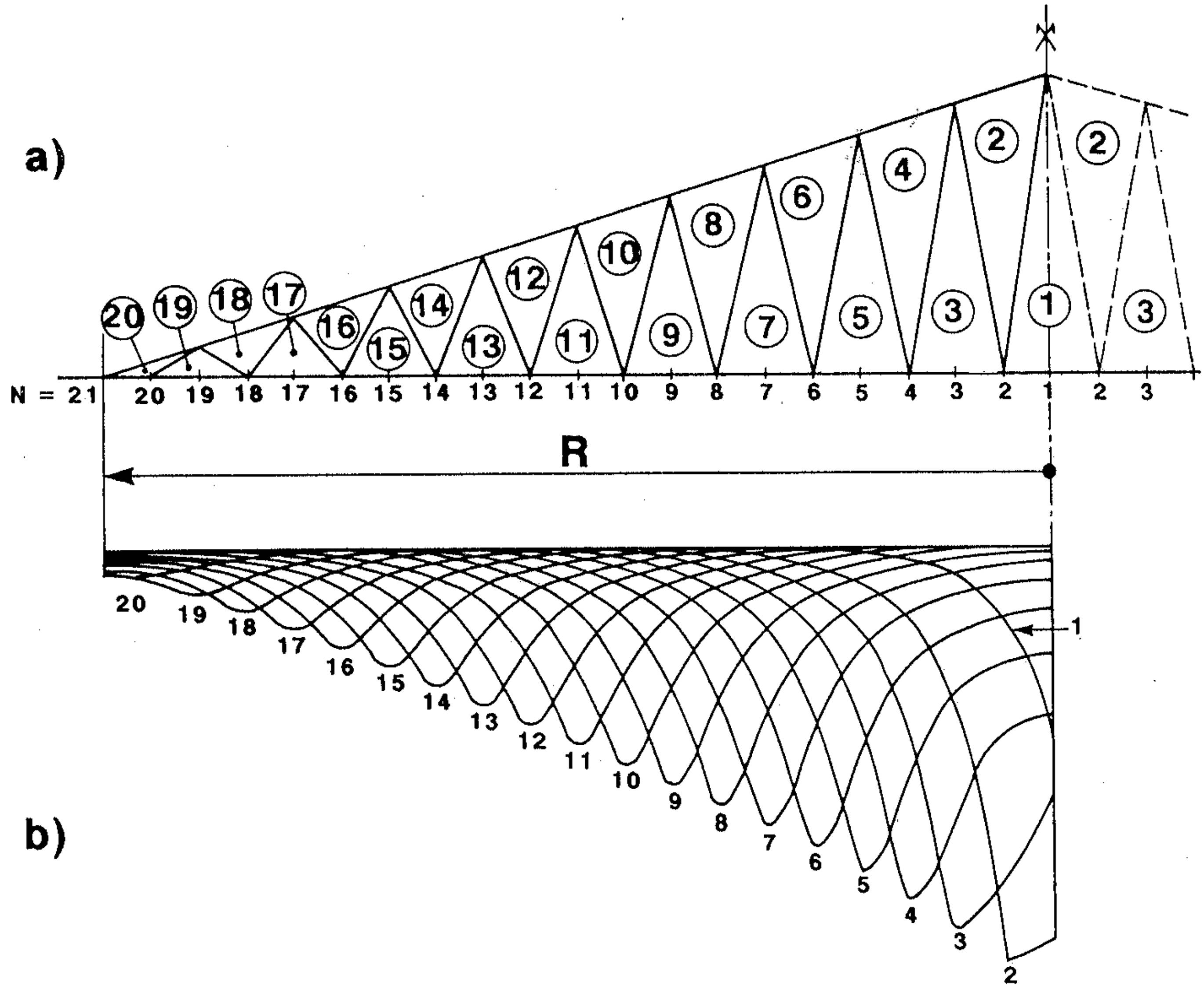


Figure 7. Example of a set of 20 unit loads: (a) unit loads; (b) corresponding deformation of a rock layer 0.3 times as thick as the distance *a* of the contact points.

Using the values ϑ_{IJ} of the 'local settlement' as coefficients of a matrix, and considering the initial shape of the top is defined by $U(r)$ or U_k, a system of linear equations can be set up quite easily, ensuring that, from the centre to the given limiting radius R_L the levels of the various points of the deformed cylinder's top are exactly the same. This condition expresses itself in the following series of linear systems of equations. For each, $L = 2$ to n. L gives the extension of the contact zone increasing from zero, $L = 1$, to the total surface, $L = n$ (for example $n = 21$). The linear system is the following:

$$V_I = U_I - \sum_{J=1}^{L-1} \vartheta_{IJ} \cdot p_J = V_L \quad \text{for each, } I = 1 \text{ to } L \tag{2}$$

U_I being the initial 'elevation' of the point I

V_L is the final 'elevation' of all the points defined by $I \leq L$ (see Figure 5)

ϑ_{IJ} is the deformation of the upper surface at point I due to the unit load at point J (depending on the ratio of the thickness of the rock layers d to the distance a of the contact points, see Figure 6).

p_J is the acting force (pressure) at point J (obviously at the boundary R_L, $p_L = 0$).

$F_L = \sum_{J=1}^{L-1} C_J \cdot p_J$ is the total force necessary for the contact zone to reach the radius R_L, where C_J is the force corresponding to the unit load p_J.

$S_L = U_1 - V_L$ represents the total 'settlement' starting from the first contact.

$\alpha_L = (\frac{L-1}{n-1})^2$ is the ratio of the contact zone to the total surface, that is, the 'ratio of closure'.

In solving all the linear systems of equations, one obtains n values of the total force F_L, and therefore of the total stresses

$$\sigma_{t_L} = \frac{F_L}{\pi R^2}$$

as well as the corresponding n values of the 'settlements' S_L, and therefore of the strains

$$\varepsilon_L = \frac{S_L}{d/2}$$

Figure 8 can then be drawn, point by point; it shows the strain-stress relationship for the fissured, elastic, dry rock mass. The section D-O represents the condition of no contact between the two blocks; at point O the contact is established; O-A is the progressive closing of the joints and A-B the linear elastic deformation of the rock mass after the complete closure of the joints.

Indeed the method followed in the computations shown above is simply the usual one needed to describe the behaviour of an elastic structural system which changes its shape (joint closure) during the loading process. This leads, obviously, to a non-linear elastic body law.

4.2 Influence of the neutral pressure

The water present in the open joints does not have any effect on the stresses nor the strains, unless the joints are completely filled and the water is pressurized. The influence of this pressure p, according to Figure 1, is very easily understood and described if one increases the total stress σ_t by the same amount, maintaining constant the effective stress σ_e. The ratio α does not change as shown in Figure 9.

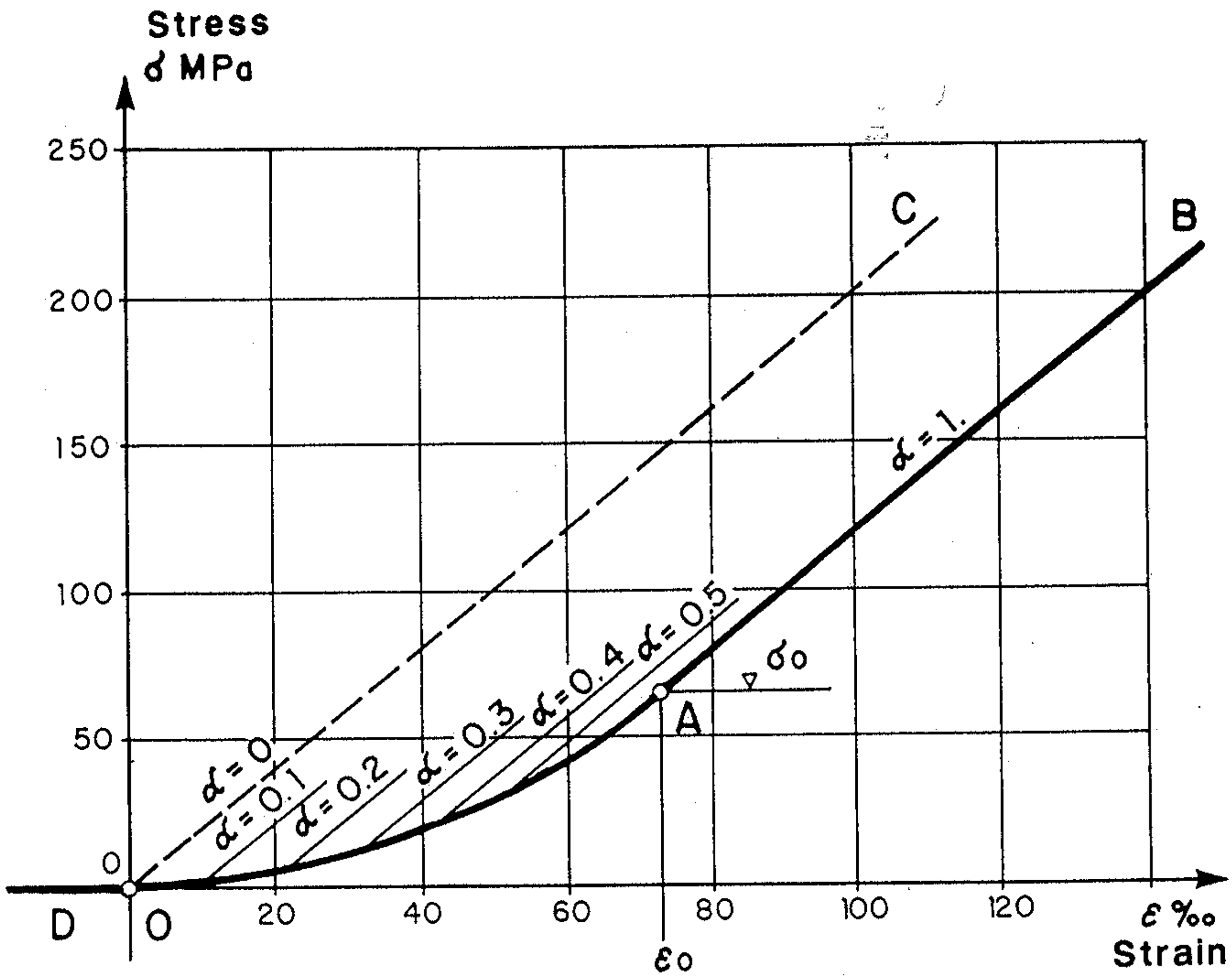


Figure 8. Example of a stress-strain relationship for a dry fissured rock mass (closing curve): α = closing ratio.

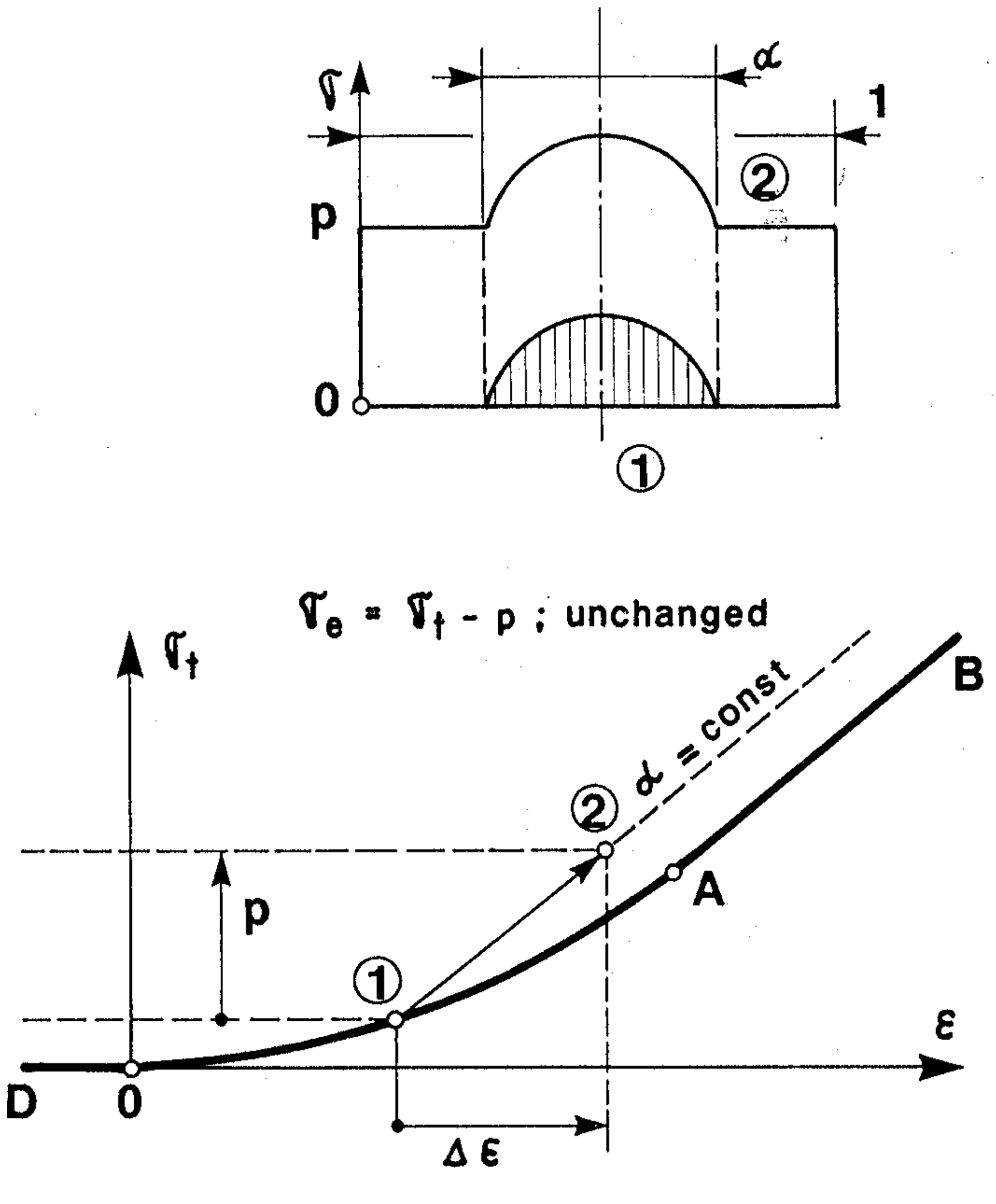


Figure 9. Introduction of an interstitial (neutral) water pressure p in a fissured rock mass: the representative point is displaced from (1) to (2).

The pressure p (or the pressure change from zero) displaces simply the representative point of the stress-strain state from state (1) to state (2) where, taking into account the lateral constraint

$$\Delta\varepsilon = \frac{p}{E} \cdot \left(1 - \frac{2\rho^2}{1-\rho}\right) \quad \text{and} \quad \sigma_{lat} = p \cdot \frac{\rho}{1-\rho} \tag{3}$$

as in fact an additional uniformly distributed load p simply acts on the entire surface of the joint. Varying the pressure p in a parametric way, the entire field of the uniaxial FES model can be set up and drawn.

5 The FES model

On this basis the FES model, as seen in Figure 10, was plotted (corresponding to the case of uniaxial strain). Each point of the plane on top of the line DOAB represents a stress-strain state of the rock mass which relates together four values; or more precisely:

- the total stress σ_t
- the neutral pressure p
- the strain ε, and
- the ratio of closure α

On the left of the line O-C, any point corresponds to an entirely open joint, or gap, without any contact of the two rock blocks, but where an interstitial water pressure (equal to the total stress) may be acting. Such a point can be arrived at, for example, during a grouting process when a 'claquage' takes place and also in the case of hydro-fracturing.

Starting from any point on the graph, that means from any stress-strain state, changes may take place in any direction; such changes of state can be represented by a vector.

However, four directions deserve special attention; they are shown in Figure 11 by four quite interesting arrows.

1. The first remarkable variation takes place in direction (1). This is simply a loading of the rock mass in undrained conditions. The water cannot escape. Assuming, as a matter of simplicity, an adequate compression of the water, this direction is parallel to A-B and the ratio of closure α remains constant. The apparent modulus of elasticity is

$$E^* = \frac{E \cdot (1-\rho)}{1 - 2\rho^2 - \rho} \tag{4}$$

2. The second remarkable variation of the stress-strain state (2) maintains the neutral pressure constant while the total stress changes. The apparent modulus of deformation is MD.

3. The third remarkable direction (3) corresponds to a change in the internal pressure while the total stress is kept constant. This case occurs, for example, when draining a rock mass by constant overburden (as in the case of Zeuzier). The 'modulus of settlement' is defined by MT.

4. The fourth remarkable direction (4) corresponds to simultaneous variations of the total stress and the water pressure in such a way as to avoid any change in strain. The ratio of the variations of the two values, total stress and neutral pressure, indicated by ς, is an additional characteristic of the behaviour of the rock mass.

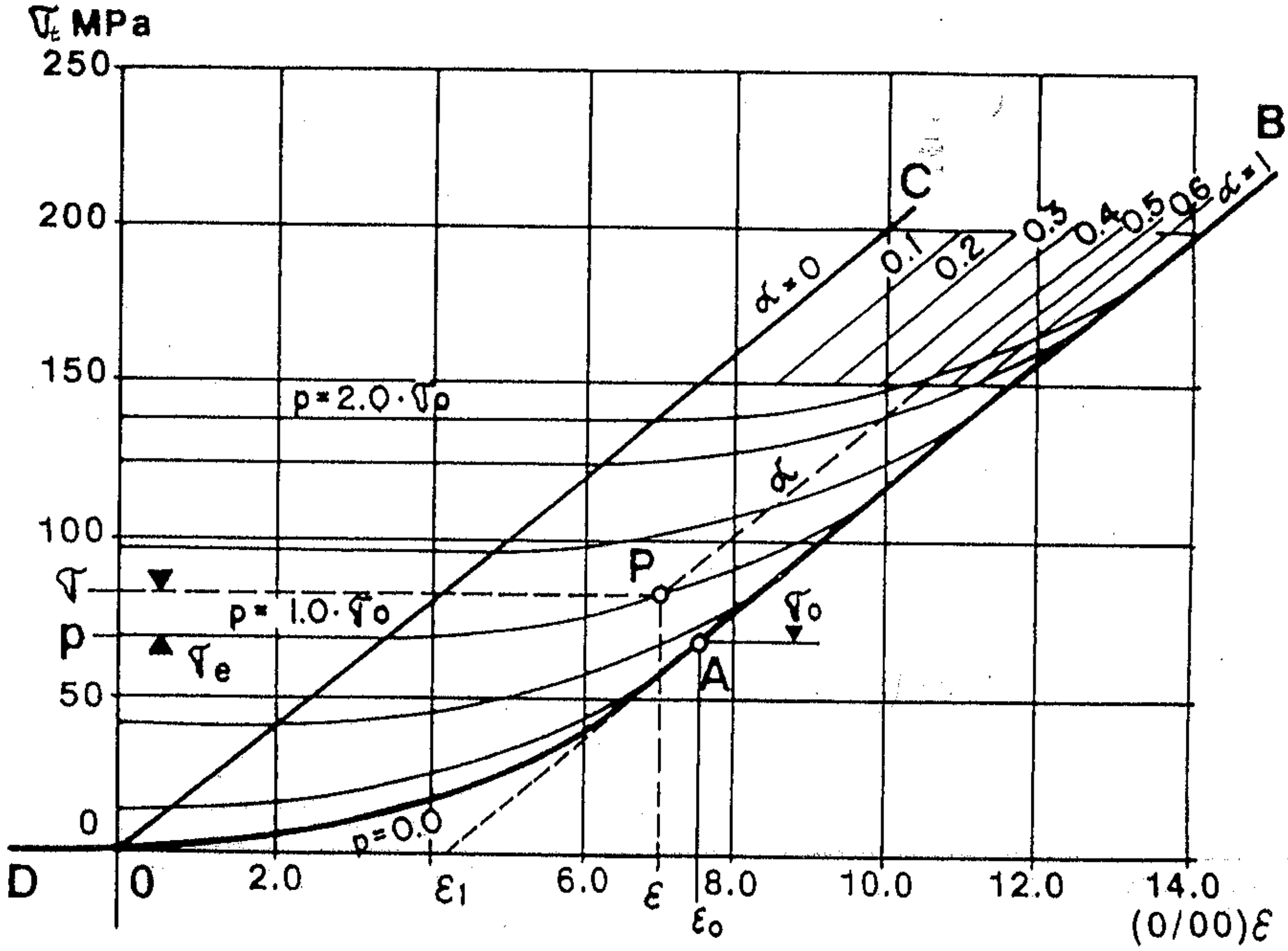


Figure 10. Example of an FES model of a rock mass: ε = strain; σ_t = total stress; p = neutral water pressure; α = degree of closure (α = 1 fissures completely closed, α = 0 fissures completely open); A $(\varepsilon_o, \sigma_o)$ = point of total closure at nil water pressure; σ_e = effective stress; P $(\varepsilon, \sigma_t, p, \alpha)$ = general strain, total stress, neutral pressure and closing ratio at point P.

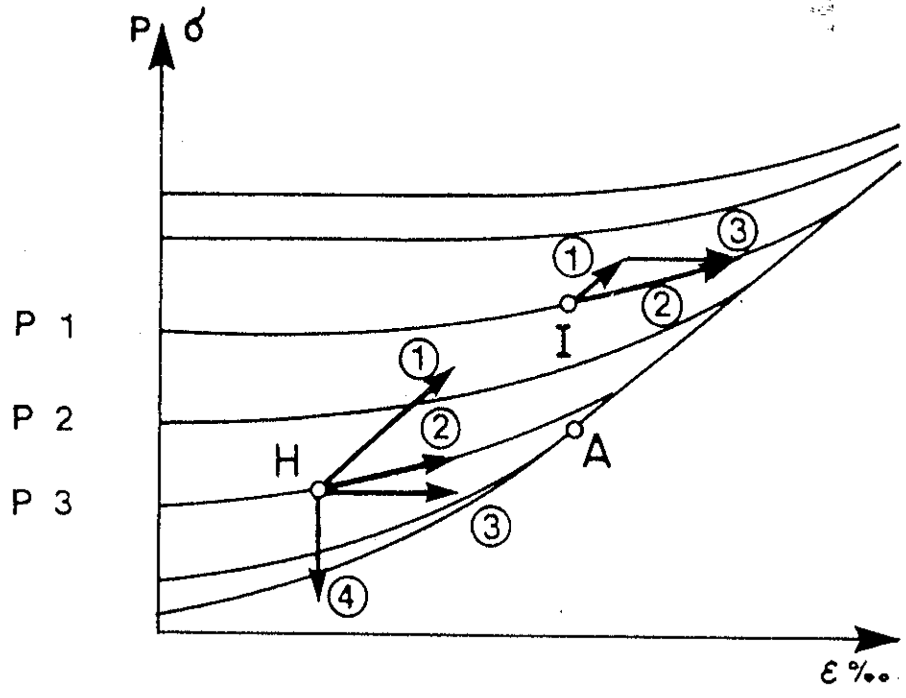


Figure 11. FES model – significant changes of the stress- strain state: (1) undrained loading; (2) loading at constant water pressure; (3) drainage at constant load; (4) change of state by no variation of the strain.

The definition of these moduli and of this ratio, as well as the very meaningful relationships existing among them, are summarized below. They allow an understanding of the behaviour of the fissured rock mass. It needs to be mentioned that these values are not constant but variable from point to point. They are functions of the state of stress and strain.

Definitions valid for any approximation formula

$$MD = \frac{\partial \sigma}{\partial \varepsilon} \quad (\partial p = 0)$$

$$MT = \frac{\partial p}{\partial \varepsilon} \quad (\partial \sigma = 0)$$

$$\varsigma = \frac{\partial \sigma}{\partial p} \quad (\partial \varepsilon = 0)$$

$$E = \frac{\partial \sigma}{\partial \varepsilon} \quad (\partial \sigma = \partial p) \quad (\partial S = 0)$$

Relationships valid for any approximation formula

$$\frac{1}{E} = \frac{1}{MT} + \frac{1}{MD}$$

$$MD = E \cdot (1 - \varsigma)$$

$$MT = E \cdot (1 - \frac{1}{\varsigma})$$

$$MD = -\varsigma \cdot MT$$

where:
E = Modulus of elasticity (or $E \cdot (1-\mu)/(1-\mu-2\mu^2)$)
MD = Modulus of deformation
MT = Modulus of settlement
ς = Ratio of total stress against water pressure
σ_t = Total stress
p = Neutral water pressure
$S = \sigma_t - p$ = Effective stress
ε = Strain

6 Approximation of FES

As defined up to now, the FES model is a pure numerical one. To make its use more simple, a number of analytical approximations were set up. Three of them, called FESA, FESAB and FESEX, are based respectively on a one term, on a two term polynome and on an exponential function. They are explained below.

Definitions for three approximations of FES

FESA: $(\frac{\sigma-p}{\sigma_o}) = (\frac{\varepsilon-p/E}{\varepsilon_o})^{\eta}$; $\eta = \varepsilon_o \cdot E/\sigma_o$

$\xi = (\frac{\sigma-p}{\sigma_o})^{\frac{\eta-1}{\eta}} = (\frac{\varepsilon-p/E}{\varepsilon_o})^{\eta-1}$

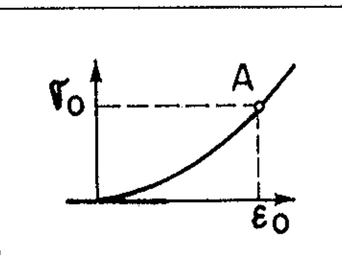
FESAB: $(\frac{\sigma-p}{\sigma_o}) = A(\frac{\varepsilon-p/E}{\varepsilon_o})^{m} + B \cdot (\frac{\varepsilon-p/E}{\varepsilon_o})$

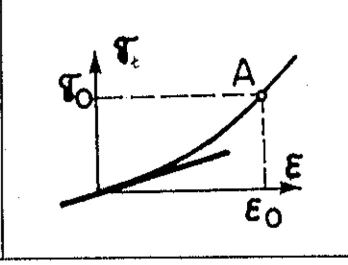
$\underset{m=1+\frac{\eta-1}{A}}{\overset{A+B=1}{A>0}}$; $z = \frac{A \cdot m}{\eta}(\frac{\varepsilon-p/E}{\varepsilon_0})^{m-1} + \frac{1-A}{\eta}$

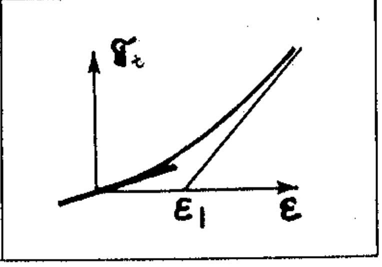
FESEX: $\frac{\sigma}{E} = \varepsilon - \varepsilon_1(1-e^{-k\cdot(\sigma-p)})$; $\phi = \varepsilon_1 \cdot k \cdot e^{-k\cdot(\sigma-p)}$

Relationships for three approximations of FES

Modulus/ratio	FESA	FESAB	FESEX
	for $0 \leq S \leq \sigma_o$	$0 \leq S \leq \sigma_o$	$0 \leq S$
$MD = \frac{\partial\sigma}{\partial\varepsilon}$ $\partial p = 0$	$E \cdot \xi$	$E \cdot z$	$\frac{E}{1+E\cdot\phi}$
$MT = \frac{\partial p}{\partial\varepsilon}$ $\partial\sigma = 0$	$\frac{-E\cdot\xi}{1-\xi}$	$\frac{-E\cdot z}{1-z}$	$\frac{-1}{\phi}$
$\varsigma = \frac{\partial\sigma}{\partial p}$ $\partial\varepsilon = 0$	$1-\xi$	$1-z$	$\frac{\phi\cdot E}{1+\phi\cdot E}$
$E = \frac{\partial\sigma}{\partial\varepsilon}$ $\underset{\partial\sigma=\partial p}{\partial S=0}$	E	E	E







where:
σ_o = Total stress at joint closure (dry rock)
ε_o = Strain at joint closure (dry rock)
η = Non-linearity number
ε_1 = Strain caused by joint closing ($\sigma_o = \infty$)
k =Closing coefficient
$S = \sigma_t - p$ = effective stress

Depending on the type of rock mass, one or the other approximation fits best with the numerical values of FES; the choice shall be done correspondingly in each case. All the approximations fulfil exactly all the relationships shown at the end of section 5.

7 Additional effects

Some additional effects were introduced later on, as extensions of the original FES model, as described above. In the case when the surfaces of the rock blocks are slightly weathered, a softer layer of matter of a given thickness can be introduced in the computation of the deformation coefficients from the unit loads. This effect is then automatically transferred to the resulting FES model.

Some crushing of the rock matter at the contact spots may take place because of high compressive effective stress. This phenomenon founds a kind of pseudo-plasticity of the fissured rock mass itself. It may easily be understood how a limiting crushing stress and a corresponding non-linear and non-reversible closing of the joints can be introduced in the FES model and how the computation technique has to be refined to do so. An example of such an 'elasto-plastic' computation is shown in Figure 12.

The FES model was originally set up for a uniaxial acting stress and a uniaxial strain pattern with completely restrained lateral displacements. It corresponds very well, for example, to the case of horizontal rock layers or beds separated by parallel joints or fissures and is subject to a vertical load in the presence of a ground water table.

The cases an engineer has to deal with are obviously, as a rule, far more complicated. More than one single regularly spaced system of joints exist; each one with its own orientation in space and particular characteristics.

Also the condition of total lateral constraint needs to be released. Obviously, an identical water pressure in all joints, at any point in the rock mass, regardless of their orientation has to be taken into account.

Handling such a complex system with precision may not always be an easy task. Fortunately, however, the orthotropic case with three joint systems placed orthogonally to each other, even with different characteristics, can be dealt with by iteration in a relatively simple manner. This configuration may apply to a great number of the actual cases the engineer is faced with.

8 Hydraulics

Another extremely relevant problem in rock mechanics is represented by the hydraulics of the fissured rock mass. If the porosity and permeability of the rock matrix itself is discarded, the main question is then the flow of water – or grout – which can take place along joints, fissures or any kind of discontinuities. also the porosity, which corresponds to the volume of voids of the discontinuities, needs to be studied.

Figure 13 provides the bases for the computation of both porosity and permeability in the plane of a joint according to the definitions given above. The ratio of closure α and therefore the effective stress σ_e are to be considered constant during the computation, but may obviously change with time and can assume different values; porosity and permeability are clearly functions of the state of stress.

A system of 'channels' around the hexagonal contact spots or 'islands' allows the fluid to progress along the joint under the effect of a given gradient of pressure. For the sake of simplicity, only laminar flow will be investigated in the following.

In fact the hydraulic system is represented by a net of short channels of assumed constant section and length, three of which connecting at each time.

It can easily be proven that with the above configuration the permeability in the plane is, for practical purposes, the same regardless of the overall direction of the pressure gradient. It is therefore justified to compute only the case of a gradient coinciding with one of the directions of the channels. The specific flow q in the joint for a unit width is

$$q = \frac{Q}{a} \tag{5}$$

where Q is the flow in a single channel.

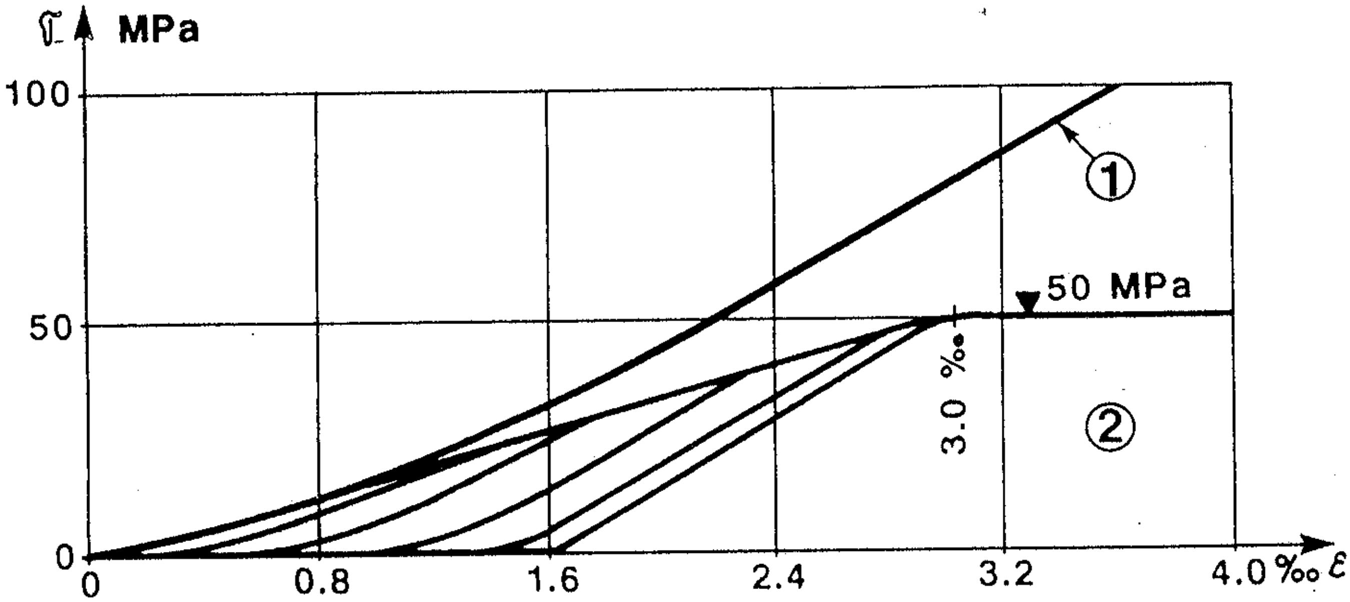


Figure 12. FES model applied to Kölnbrein arch dam, shale No. 3 (dry rock): σ_t = total stress; ε = strain; (1) elastic case; (2) elastoplastic case

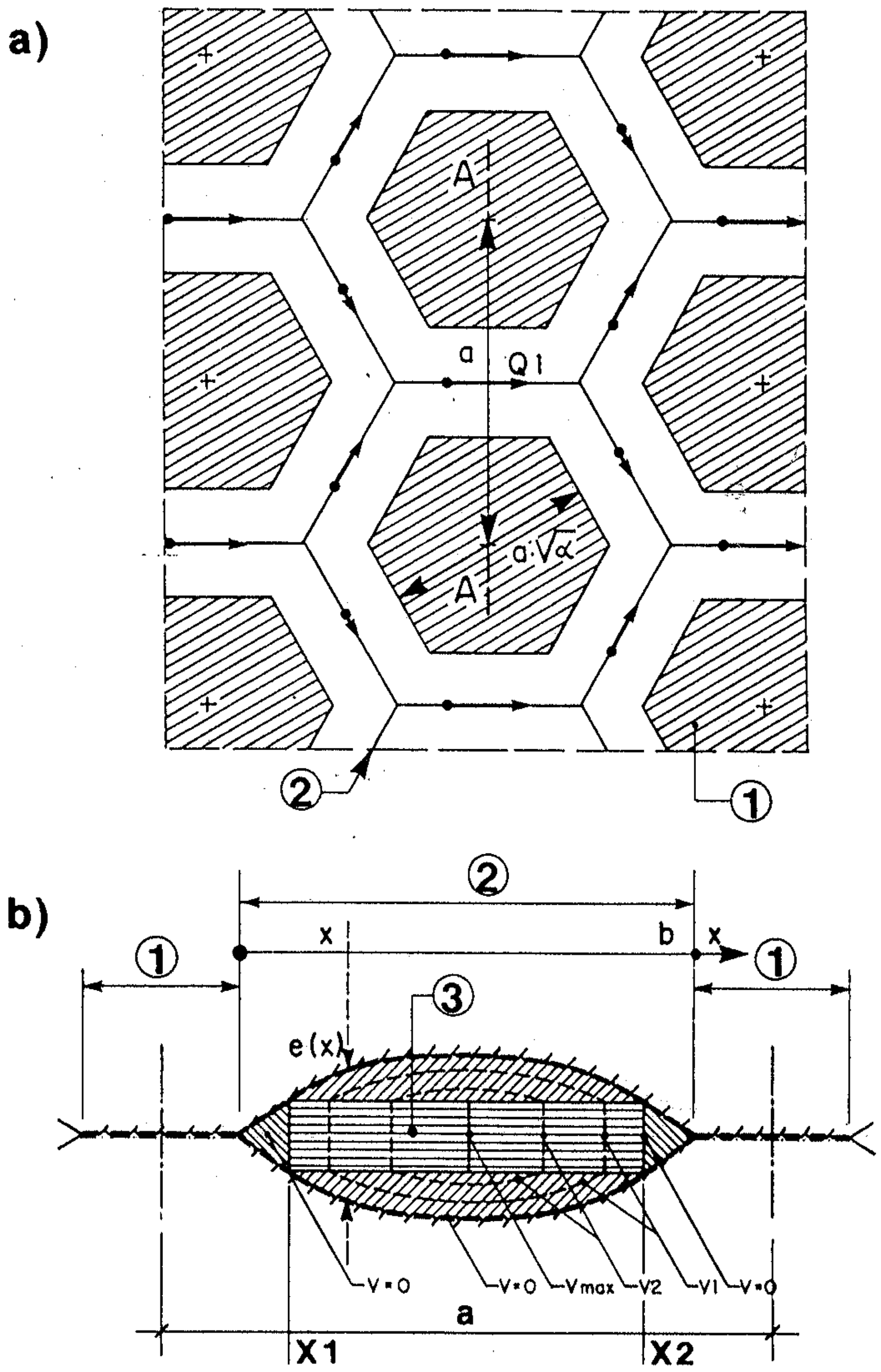


Figure 13. FES model of a fissure along its plane (for hydraulic mathematical analysis): (a) pattern of channels and islands; (b) cross section of a channel; Q_1 = flow through a channel; (1) contact zones; (2) flow paths; (3) semi-rigid core (in the case of a cohesive fluid); v = flow velocity in the case of a Bingham's grout.

At the confluence, the flow Q splits in two parts, which in this special case are equal. The pressure gradient J in the channel AB equals the overall gradient, while in the following branches it equals just half of this value.

The flow in the single channel as a function of the gradient can then be found (see Figure 13b). The flow induced by the gradient J can be obtained by integrating across the section of the channel as a function of the thickness e of the opening.

In the case of a Newtonian fluid, like water, with a viscosity η the flow is simply:

$$Q = \frac{J \cdot \gamma}{12 \cdot \eta} \cdot \int_o^b e(x)^3 \cdot dx \tag{6}$$

where γ is the specific weight of the fluid.

In the case of a fluid with viscosity η and cohesion c, like cement grout - which is a Bingham's body - the computation is somewhat more complicated, as a minimum thickness ($e_{\min} = \bar{e}$) is required to permit the grout to flow at a given gradient (Lombardi [1985]).

$$\bar{e} = e_{\min} = \frac{2 \cdot c}{\gamma \cdot J} \tag{7}$$

The grout will therefore flow only in the central part of the channel while it will stick at both borders (Figure 13b). Additionally the flow in a single element of the channel width is not any longer a linear function of the gradient, but has to comply with the following condition:

$$dq = \frac{\gamma J \cdot e^3}{12 \cdot \eta} \cdot \left(1 - \frac{3}{2} \cdot \frac{\bar{e}}{e} + \frac{1}{2} \cdot \left(\frac{\bar{e}}{e}\right)^3\right) \cdot dx \; ; \quad \text{for } e \geq \bar{e} \tag{8}$$

So the total flow is

$$Q = \int_{x=x1}^{x2} dq \tag{9}$$

In the case of a cement grout, the size of the grains – in fact the dimension of the grain flocks – has to be taken into consideration in relation to the thickness of the opening at any point. This fact reduces very substantially the flow of the grout mix along the joint (Lombardi [1989]).

The method used to compute the volume of the voids, that is the porosity of the rock mass, is not detailed here.

The considerations given above allow for the direct computation of the porosity and the permeability of the rock mass as well as the penetrability into the joint of a given grout mix. Accordingly, a number of significant relationships can be established.

However, it is felt that, in the case of permeability and penetrability, the FES model is likely to represent an upper bound, as certain disturbances in the flow at the channel confluences and at the narrow sections of the channels, as well as the effect of widening and narrowing the channels, were not considered. Nevertheless the results proved to be useful, first of all in order to understand the influence of the parameters involved.

As for the FES model itself, analytical fittings, to be given parameters in each individual case, were developed both for porosity and viscosity as functions of the effective stress or the closing ratio. An example of the influence of the effective stress on permeability and porosity is shown below[3].

PERMEABILITY:

$$k = k_o \cdot (1 - \frac{\sigma_e}{\sigma_o})^{m_1} \; ; \; k_o = 2.5 \cdot 10^{-4} \text{ m/s}$$
$$m_1 = 10$$

POROSITY:

$$n = n_o \cdot e^{-m_2 \sigma_e / \sigma_o} \; ; \; n_o = 6.5 ‰$$
$$m_2 = 5.2$$

TOTAL JOINT CLOSURE AT:

$$\sigma_o = 55 \text{ MPa}$$
$$\varepsilon_o = 9.42 ‰$$

9 Some applications

On the basis of the theory presented above a comprehensive computer program was developed and a number of cases were analysed; some of them are presented in the following figures.

A first factor studied is the influence of the shape of the undulation of the joint opening on the stress-strain curve for dry rocks. In Figure 14, five cases are represented which show this influence quite clearly.

A second factor of great importance for the engineer is the variation of the rock mass permeability as a function of the state of stress and the joint opening of the joints at no load. An example is given in Figure 15.

The influence of the grain size of the cement grout on the penetrability can be seen in Figure 16. It follows from this figure also, that no general simple relationship can exist between the results of Lugeon – or any other water pressure test – and the cement quantity taken at grouting time.

A complete application of the FES model was done, as already mentioned, in the case of the very exceptional settlements at the Zeuzier arch dam. These results will be presented and discussed in Part 2 of this paper.

10 Limitations

As with any mathematical tool, the FES model can obviously not be more than an approximation of the very complex behaviour of an actual rock mass; the approximation being of variable quality depending on the case being studied.

In addition, it has to be clearly understood that FES is intended to represent the body law of a fissured, elastic, saturated rock mass subject to compressive stresses and their variations. By no means can it be used in cases of excessive shear stresses along the joints and fissures; this would produce slippage causing a change in the fitting of the rock blocks in contact at the joints and possibly also some dilatancy of the rock mass. There may also exist cases of clay filled joints which may be better investigated in using other mathematical tools.

[3] FESA approximation for Zeuzier rock-mass (Dogger formation) – one set of joints

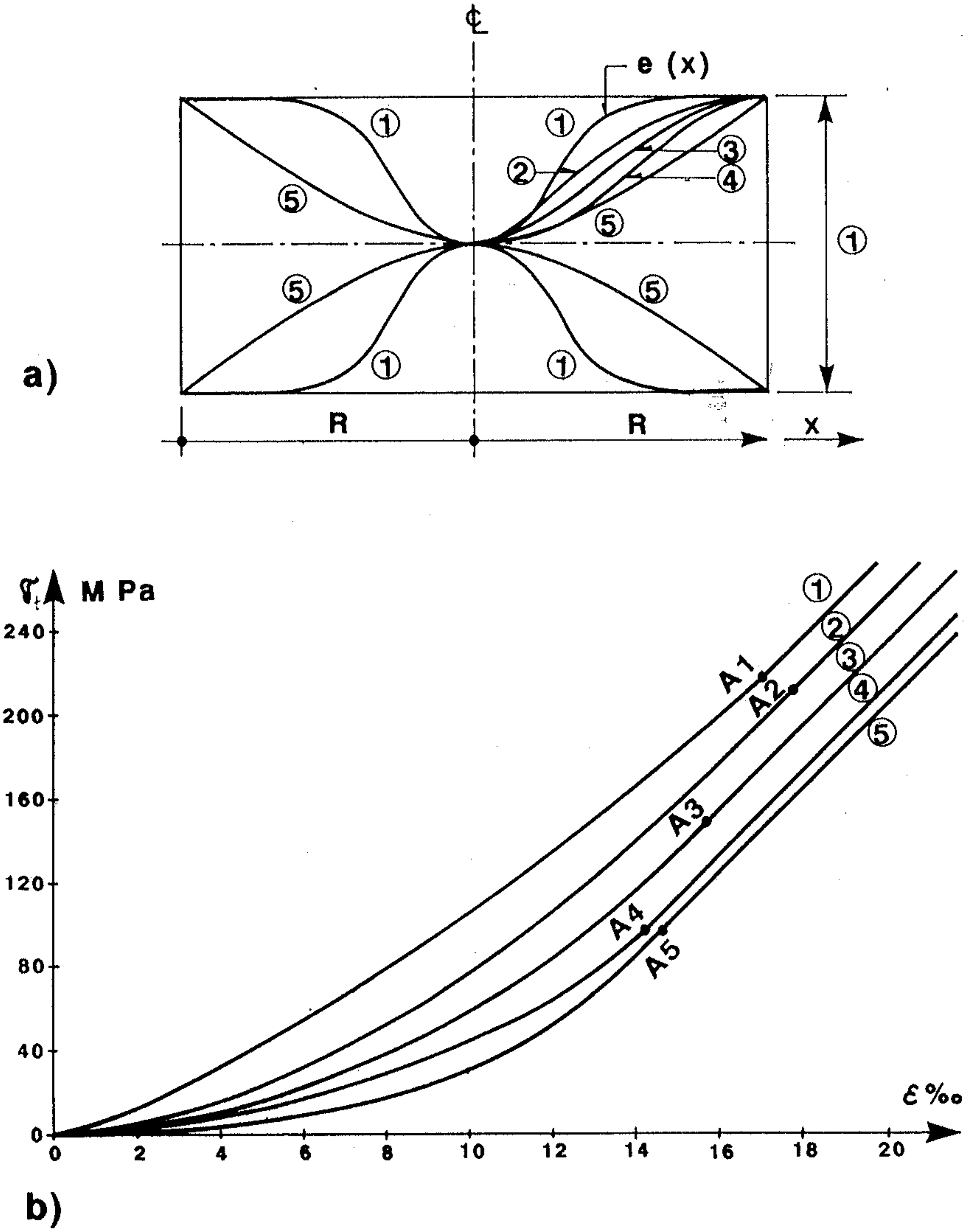


Figure 14. Example of the influence of the shape of the joint surface on the stress-strain curves: (a) shapes of the opening; (b) corresponding stress-strain curves for dry rock for the same maximum opening and the same distance of the joints; (1) joint thickness = maximum opening.

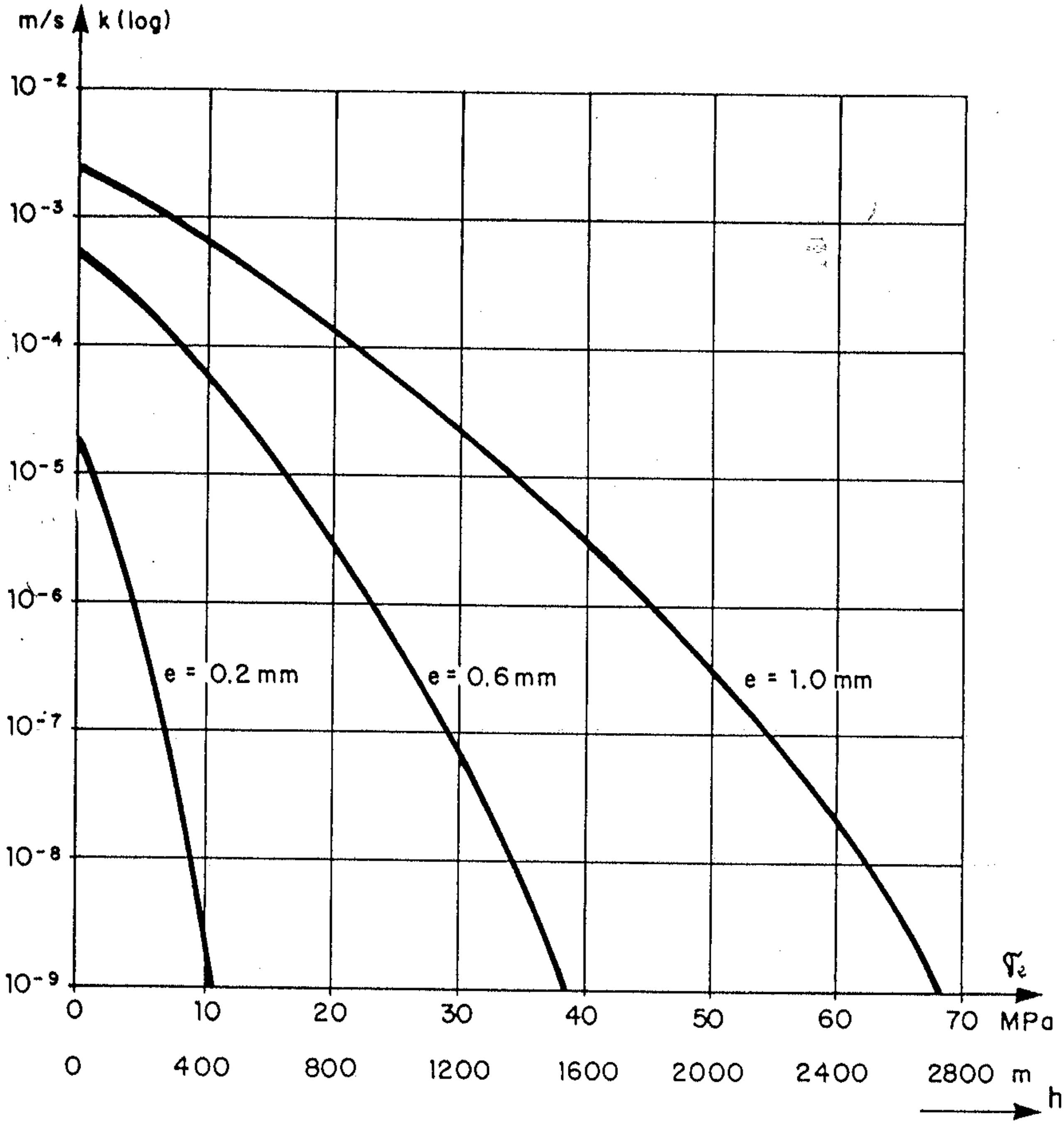


Figure 15. FES model – permeability of the rock mass as a function of the effective stress: example shows different joint opening (undulation) values; k = permeability; σ_e = effective stress; h = depth below ground.

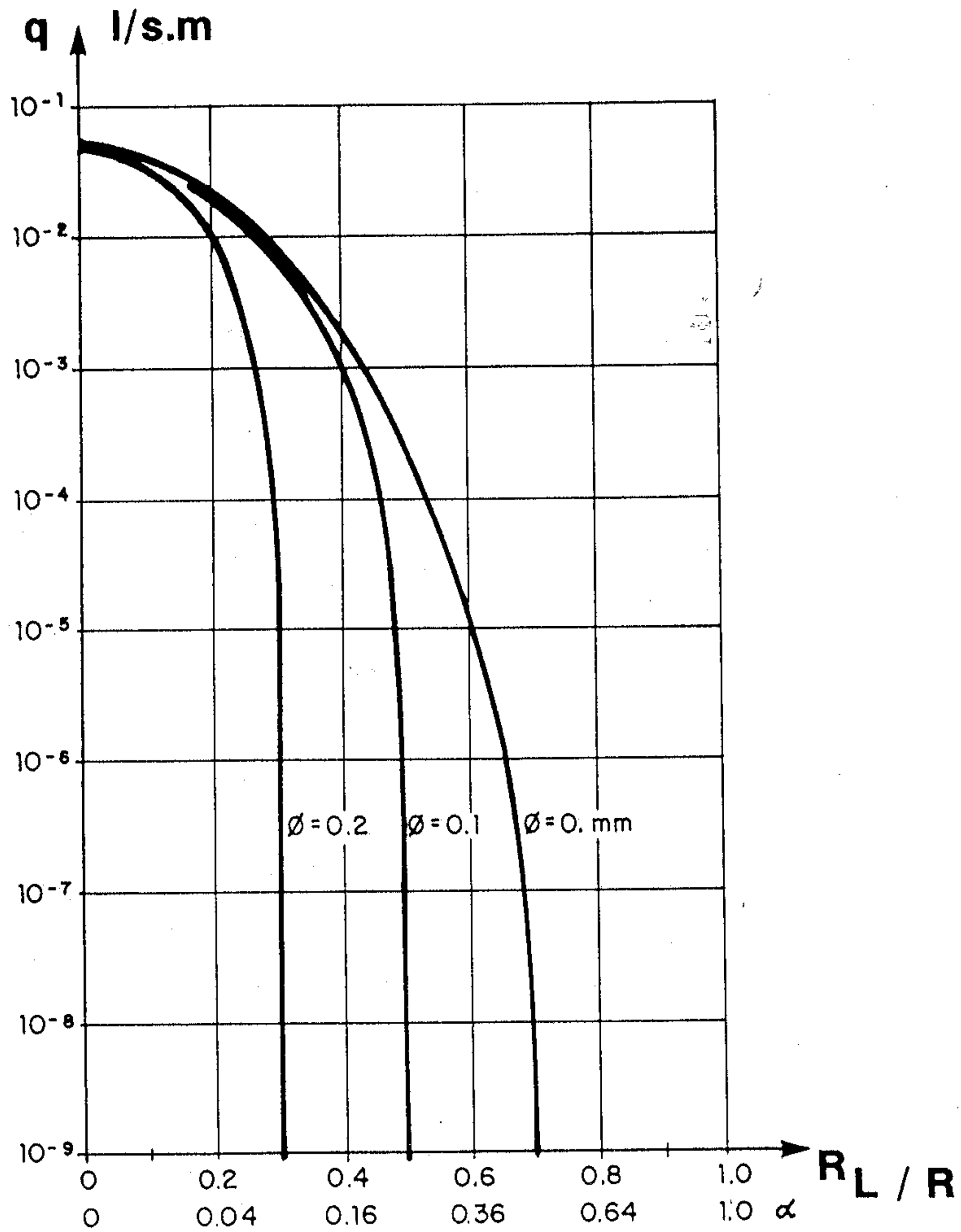


Figure 16. **Effect of maximum grain size of cement particles (or flocks) on the penetrability of a grout as a function of contact radius R_L or of the ratio of closing α:** example shown here is for a gradient of 20.

Conclusion

In spite of the limitations mentioned here, the FES model proved to be a very useful tool placed in the hands of rock mechanics engineers.

It is felt that rock foundations for concrete dams represent a field of special interest where the FES model could be used, as slippages along discontinuities should hopefully not take place there.

An outstanding aspect appears to be the introduction in the analysis of not only deformations resulting from some estimated 'modulus of elasticity' but of deformations which actually take into account their nonlinearity as well as the influence of the changes of the interstitial water pressure.

There is no doubt, however, that further improvements of the FES model, to include additional features, are possible; more theoretical and experimental research as well as exact back analysis of case histories are suggested.

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