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GROUND REACTION TO DEEP DRAINING TUNNELS

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Knowledge of the elastic induced stresses and deformations is the first step in computing the rock elasto-plastic reaction to deep tunnel excavation. The following steps are well defined but it is the first step that, in permeable rocks, is unlikely to be satisfied. The reason is that the closed elastic solution suffers logarithmic and higher order divergences. Unlike tunnels deep caverns do not present any diverging phenomena and the paper will focus on the tunnel problem. A renormalization of seepage forces is necessary to eliminate diverging effects. Drainage reduces cohesion leading to an early failure, a greater radius of the elasto-plastic interface and a larger ground reaction. A procedure that is compatible with analytical or numerical treatments is used to compute ground reaction which is useful for tunnel design and lining dimensioning. Tunnel deformation or ground reaction is defined by two variables that are the support pressure and the water inflow.

Keywords: Seepage forces; Water inflow; Lining; Tunnel; Cavern; Plasticity.

1. Introduction

Deep tunnels or caverns are schematized as apertures in an infinite rock mass. The infinite rock mass hypothesis simplifies the evolution equations of cylindrical and spherical excavations. In impermeable rocks, closed solutions have long been used for tunnel design. They provide realistic values of the generated stresses and deformations when tunnel depth exceeds six times the elasto-plastic interface radius. This idyllic situation is no longer valid for tunnels in permeable rocks. A logarithmic divergence occurs and the solution is unbounded. The asymptotic behavior of the main variables in permeable rocks is shown in Table1 which shows that caverns are not affected by the divergence phenomenon. There are a number of potential solutions to the tunnel problem. One of these is to consider a limited extension of the rock medium with an outer cylindrical boundary at a great distance from the tunnel. Such an alternative is nothing new and can be treated developing a few lines of numerical code or by using an appropriate commercial numerical code. The scope of the paper is to resolve the logarithmic divergence. Analysis of the divergence phenomena requires knowledge of the elastic part of the solution between two concentric cylinders. The radius of the outer cylinder will then be increased indefinitely while introducing the necessary conditions to impede any diverging effect.

Table 1. Asymptotic radial behavior of the main variables in an infinite permeable rock mass.

	Cavern	**Tunnel**
Mechanical stress	r^{-3}	r^{-2}
Mechanical displacement	r^{-2}	r^{-1}
Pressure gradient	r^{-2}	r^{-1}
Water Pressure	r^{-1}	Log r
Water induced displacement	r^{0}	r Log r

2. Radial Steady State

Water percolation produces a mass force that is felt throughout the rock masses. The steady radial equilibrium equation is transformed into

$$\frac{d\sigma_r}{dr} + m\frac{\sigma_r - \sigma_t}{r} = f \tag{1}$$

in which σ_r, σ_t, f and r are the principal effective radial and tangential stresses, percolation force and radial coordinate; Parameter *m* takes on the value one and two for tunnels and caverns respectively. Percolation force is the opposite of Darcy's driving force

$$f = \frac{dp}{dr} \tag{2}$$

in which p is water pressure. There is an alternative mode to express the percolation force considering the total volume of water Q that flows across a surface of area S enveloping the tunnel or cavern and at a distance r from the center. In the case of a cavern the surface is closed and in the case of tunnel it will be cylindrical of unit longitudinal length. The percolation force is transformed considering Darcy's law and is obtained as

$$f = \frac{Q}{Sk} \tag{3}$$

in which k is the hydraulic conductivity. Seepage forces decrease as the distance from the tunnel or cavern increases because Q is a constant and S increases. For a tunnel seepage forces decreases as the inverse of the radial distance and for a cavern it decreases as its square.

2.1. *Boundaries and interface*

Three concentric surfaces with radius a, b and c will be used in the analysis; a is the radius of the tunnel or cavern, b is the radius of the elastic plastic interface and c is the radius of an outer boundary. The difference between b and a is the extent of the plastic zone. If rock unloading is not enough to produce a plastic zone a and b are equals. The difference between c and b is the extent of the elastic zone. c is assumed to be always greater than b.

2.2. *Initial and boundary conditions*

Homogeneous states of stress and water pressure are present everywhere before excavation. They are noted σ_c and p_c. These remain unchanged after excavation on the outer boundary. On the tunnel edge a radial load σ_a and a water pressure p_a are applied. These conditions are necessary to solve and find stresses and deformation at any radial distance. On the elastic plastic interface radial stress and water pressure are noted σ_b and p_b. The radial effective stress on the tunnel edge σ_a is also known as the applied effective pressure and support pressure.

2.3. *Materials*

Different linear elastic parameters, hydraulic conductivities and plastic yield criteria are admitted in the elastic and plastic zones. A non-associative plastic flow potential is also admitted. Where necessary, material parameters are indicated by superscripts e or p to distinguish the elastic and plastic zones.

2.4. *Sign convention*

Compressive stresses are negative. Water pressure is positive. Water inflow is positive for draining tunnels. Displacement is positive outward.

3. Solution Procedure

The elastic-plastic solution can be computed numerically or analytically. The choice of the yield criterion and plastic flow are determinant and can favor one treatment over the other. The resolution method is not the most important thing. A correct result is the most important thing despite the fact that different resolution methods should be compared to one another. The adopted resolution, whether it be numerical or analytical, will follow different procedures. The procedure that is used here is an adaptation for permeable rocks of the classical procedure that can be found in many rock mechanics textbooks for circular opening in impermeable rocks. The steps are

- Define the support pressure on the tunnel edge and the amount of water inflow
- If elastic stresses have reached the failure criterion go to next step else compute the required information using the elastic solution
- Calculate the extent of the plastic zone or the elastic plastic interface by equating the elastic radial stress and plastic radial stress at the interface
- Calculate the elastic displacement at the interface using the elastic solution
- Calculate the deformation of the plastic zone considering that radial displacement is continuous across the interface
- When ground reaction is sought, compute tunnel deformation and return to the first step changing the water inflow and the support pressure

4. Beyond The Elastic Plastic Interface

Closed analytical solutions to the elastic state between two concentric surfaces in 2D and 3D are known. They can be used to compute the necessary information beyond the elastic plastic interface when the radius of the outer boundary is finite. For infinite medium these solutions will be used if increasing indefinitely the radius of the outer boundary creates a converging process. Appendix A and Appendix B contain reminders of the elastic solution in a drained rock between two concentric cylinders and spheres. It is obvious that, for a tunnel, the process of increasing indefinitely the outer radius, the solution does not converge. For cavern there is no diverging phenomenon. So in the case of tunnels, the problem needs special care. A different treatment has to be conceived.

4.1. *Renormalizing*

The origin of the divergence phenomenon in the elastic zone is the axisymmetric water flow. The main relation of the radial steady flow between two concentric cylinders is

$$Q = 2\pi k \frac{p_c - p_b}{\ln \frac{c}{b}} \qquad (4)$$

Increasing indefinitely the outer radius c can lead to two different situations. If the initial and final water pressures are not equal water inflow is zero and alternatively if water inflow is not zero the difference between the final and initial pressures is infinite. Both situations are absurdities. They are not real and are produced by a mathematical translation of a simplified schematization of reality. It is supposed that both the water inflow and the water pressure difference are simultaneously finite or equal to zero. This means that the ratio k/ln(c/b) should remain finite when the outer radius increases indefinitely. Rewriting the permeability as $k_0$ln(c/b) gives the desired result. This is just a mathematical artifice that will allow the solution to be extended to an unlimited rock medium. Keeping this in mind, the solution between two concentric cylinders that is given in appendix A with an infinite outer radius becomes

$$\sigma_r = (\sigma_c - (1+\alpha)\frac{p_c - p_b}{2})(1-\frac{b^2}{r^2}) + \sigma_b \frac{b^2}{r^2} \tag{5}$$

$$\sigma_t = (\sigma_c - (1+\alpha)\frac{p_c - p_b}{2})(1+\frac{b^2}{r^2}) - \sigma_b \frac{b^2}{r^2} \tag{6}$$

$$u = \frac{1}{(1-\alpha)\overline{E}}(\sigma_c - \sigma_b - (1+\alpha)\frac{p_c - p_b}{2})\frac{b^2}{r} - \frac{1}{\overline{E}}\frac{p_c - p_b}{2} r + \frac{\sigma_c}{(1+\alpha)\overline{E}} r \tag{7}$$

in which α and $\overline{E}$ take values that change for plane strain or plane stress problems and are defined in Appendix A using the Poisson coefficient ν and the elastic modulus E. The initial state is obtained when the inner stress and water pressure are equal to the outer stress and water pressure respectively

$$\sigma_r = \sigma_c \tag{8}$$

$$\sigma_t = \sigma_c \tag{9}$$

$$u_0 = \frac{\sigma_c}{(1+\alpha)\overline{E}} r \tag{10}$$

4.2. *Interface values*

At the elastic plastic interface, which is the inner boundary of the elastic zone, the radial stress and the displacement difference to the initial displacement are obtained from (5) to (7) and (10)

$$\sigma_r(b) = \sigma_b \tag{11}$$

$$\sigma_t(b) = 2\sigma_c - \sigma_b - (1+\alpha)(p_c - p_b) \tag{12}$$

$$u(b) - u_0(b) = \frac{1+\nu}{E}(\sigma_c - \sigma_b - p_c + p_b) \tag{13}$$

These three relations contain six unknowns. Other relations are needed to compute them all.

5. Failure

A yield criterion divides the stress space into allowed and forbidden regions. Inside the allowed region, the rock behaves elastically and, on its boundary, the rock behaves plastically. Mathematically, the allowed region is convex and is transcribed using an inequality that becomes an equality only when rocks enter plasticity. Plastic zones are characterized by the following yield relation

$$F(\sigma_r, \sigma_t, \kappa) = 0 \tag{14}$$

in which κ is a variable that controls the amount of hardening or softening. Locally in the stress space the yield criterion can be expanded in a first order Taylor series taking a Mohr-Coulomb shape

$$\sigma_t = \lambda \sigma_r + (1-\lambda)c \tag{15}$$

in which c and λ are a rearranging of the expansion coefficients. They are related to the Mohr-Coulomb cohesion C and friction angle φ through the following relations

$$\lambda = \frac{1+\sin\varphi}{1-\sin\varphi} \tag{16}$$

$$c = \frac{\cos\varphi}{\sin\varphi} C \tag{17}$$

The Taylor series expansion is helpful in understanding the role of water presence during the failure process in the elastic and in the plastic zones.

5.1. *Failure initiation*

At the elastic plastic interface the following relation is obtained, combining the Taylor expansion of the yield criterion (15) with the radial and tangential stresses (11) and (12) of the elastic side

$$\sigma_b = \frac{2\sigma_c - (1+\alpha)(p_c - p_b) - (1-\lambda^e)c^e}{1+\lambda^e} \tag{18}$$

Failure starts at the tunnel edge and then propagates. At failure initiation the elastic plastic radius is equal to the tunnel radius. Taking this in consideration into the above relation, it is deduced that the effective pressure at which failure initiates is greater when draining; i.e. $p_c > p_b$. Drainage anticipates failure.

5.2. *Plastic equilibrium*

Combining the equilibrium equation with the Taylor expansion of the yield criterion in the plastic zone and integrating near the tunnel edge gives

$$\sigma_r = \sigma_a \left(\frac{r}{a}\right)^{\lambda^p - 1} + \left[c^p - \frac{Q}{2\pi k^p (\lambda^p - 1)}\right]\left[1 - \left(\frac{r}{a}\right)^{\lambda^p - 1}\right] \tag{19}$$

In the plastic zone, the water inflow, the hydraulic conductivity and the friction angle influence the overall cohesion given by the first square brackets in (19). If an increasing radial compressive stress is required the applied effective load on the tunnel edge will have to satisfy the following condition

$$-\sigma_a \geq -c^p + \frac{Q}{2\pi k^p (\lambda^p - 1)} \tag{20}$$

The minimal required cohesion that produces an increasing compressive radial stress in the plastic zone is obtained from (20) considering a non supported draining tunnel; i.e. $\sigma_a = 0$ and $Q > 0$. The minimal cohesion increases with the ratio of the water inflow to the hydraulic permeability and was used in tunnel design by Egger et al. (1982). The extension of the plastic zone is obtained considering that radial stress is continuous at the interface. Equating at the interface the right hand sides of (18) and (19) for an infinite rock or (A2) and (19) for a bounded rock leads to an equation which solution is the extent of the elasto-plastic interface.

5.3. *Plastic displacement*

A first order Taylor series expansion in the stress space of the plastic flow potential G will lead to

$$G = \sigma_t - \lambda^g \sigma_r \tag{21}$$

in which λ^g is a rearrangement of the Taylor expansion coefficients and may be considered as the local dilatancy coefficient. The overall deformation in the plastic zone is the sum of the plastic and elastic deformation. The variation of the plastic deformation is the gradient in the space stress of the flow potential. These two statements are combined finding the local deformation equation

$$\frac{du}{dr} + \lambda^g \frac{u}{r} = \varepsilon_r^{ep} + \lambda^g \varepsilon_t^{ep} - \varepsilon_r^{ep}(b) - \lambda^g \varepsilon_t^{ep}(b) \tag{22}$$

in which ε_r^{ep} and ε_t^{ep} are the radial and tangent elastic deformations calculated in the plastic zone. The integration of (22) is straightforward leading to a closed form using expression (19) for the radial stress, the corresponding tangent stress given by (15) and the linear elastic stress-deformation relations with their corresponding plastic zone coefficients. The closed form is valid locally in the plastic zone as long as the yield function and potential flow Taylor expansions are valid approximations.

6. Ground Reaction

For permeable rocks, the ground reaction becomes a surface sustained by two variables, which are the applied effective pressure and water inflow. Figures 1 and 2 illustrate the effect of drainage on ground reaction and the state of stress. Iso-Q curves versus the effective applied pressure are used for the illustration. A positive Q is for a draining tunnel and a negative Q is for a water tunnel under pressure.

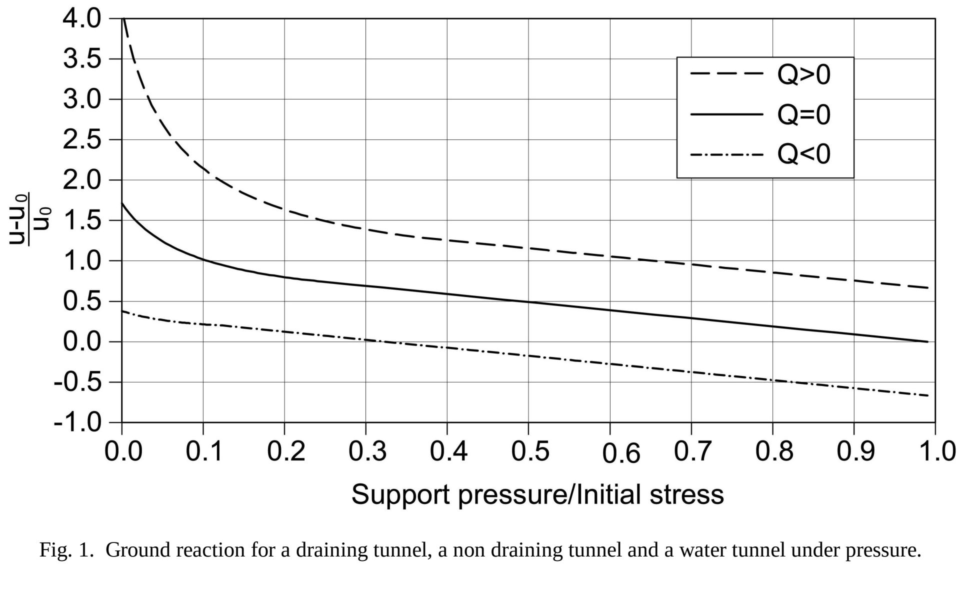


Fig. 1. Ground reaction for a draining tunnel, a non draining tunnel and a water tunnel under pressure.

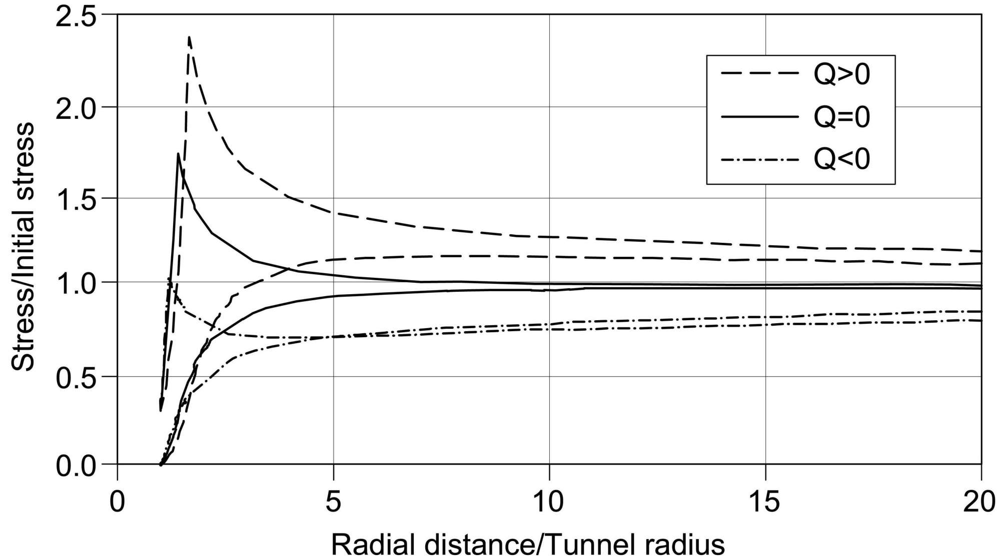


Fig. 2. State of stresses for different drainage conditions. Lower and upper curves are the radial and tangent stresses.

The ground reaction and the state of stresses change in different draining conditions. Negative water inflow corresponds to aquifer recharging with a greater pressure at the tunnel edge than the initial one. Increasing water pressure at the tunnel edge in a low permeable rock may transfer a part of it on the applied load and may lead to hydrojacking or hydrofracturing as discussed by Deere et al. (1989). Other aspects of draining tunnels are treated by Adachi (1986), Anagnostou et al. (2005), Bilfinger (2005) and Egger et al. (1982) and to non-radial flow by Fernandez et al. (1994).

7. Conclusion

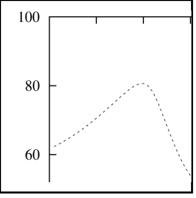
Water drainage produces in tunneling a diverging phenomenon that is overcome by redefining the permeability relation in the elastic zone. Stresses and displacements may be obtained in a closed form all over the rock mass depending on the yield and plastic flow potential that are used. First order Taylor expansions of the yield criterion and plastic flow potential allow a close insight into local drainage effect. Drainage produces a premature failure and reduces the overall cohesion.

Appendix A – Steady linear hydro elasticity between two concentric cylinders

A compact radial stress is defined using the effective radial stress and the water inflow as

$$\overline{\sigma}_r = \sigma_r - (1-\alpha)\frac{Q}{8\pi k} - (1+\alpha)\frac{Q}{4\pi k}\ln r \qquad \text{(A1)}$$

in which α is equal to ν for plane stress problem and to $\nu/(1-\nu)$ for plane deformation problem. A compact tangential stress is defined similarly to the radial compact stress (A1) changing the subscripts r to t and is distinguished from the effective tangent stress by an upper bar. From elasticity stresses and radial displacement at a distance r from the center are calculated from the



boundary radial stresses σ_b and σ_c and their corresponding compact forms, which are distinguished by an upper bar replacing in (A1) r by b and c respectively

$$\bar{\sigma}_r = \frac{\bar{\sigma}_b / c^2 - \bar{\sigma}_c / b^2}{1/c^2 - 1/b^2} - \frac{\bar{\sigma}_b - \bar{\sigma}_c}{1/c^2 - 1/b^2} \frac{1}{r^2} \qquad \text{(A2)}$$

$$\bar{\sigma}_t = \frac{\bar{\sigma}_b / c^2 - \bar{\sigma}_c / b^2}{1/c^2 - 1/b^2} + \frac{\bar{\sigma}_b - \bar{\sigma}_c}{1/c^2 - 1/b^2} \frac{1}{r^2} - \frac{(1-\alpha)Q}{4\pi k} \qquad \text{(A3)}$$

$$u = \frac{1}{(1+\alpha)\bar{E}} \frac{\bar{\sigma}_b / c^2 - \bar{\sigma}_c / b^2}{1/c^2 - 1/b^2} r + \frac{1}{(1-\alpha)\bar{E}} \frac{\bar{\sigma}_b - \bar{\sigma}_c}{1/c^2 - 1/b^2} \frac{1}{r} + \frac{Q}{4\pi k \bar{E}} r(\ln r - \frac{1}{2}) \qquad \text{(A4)}$$

in which

$$\bar{E} = \frac{E}{(1+\nu)(1-\alpha)} \qquad \text{(A5)}$$

Appendix B – Steady linear hydro elasticity between two concentric spheres

A compact radial stress is defined as

$$\bar{\sigma}_r = \sigma_r + \frac{\nu}{1-\nu} \frac{Q}{4\pi k} \frac{1}{r} \qquad \text{(B1)}$$

Radial compact stresses on the boundaries are distinguished from the effective radial stress σ_b and σ_c by an upper bar replacing in (B1) r by b and c respectively. A compact tangential stress is defined similarly to the radial one changing in (B1) the subscripts r to t. The elastic solution is

$$\bar{\sigma}_r = \frac{\bar{\sigma}_b / c^3 - \bar{\sigma}_c / b^3}{1/c^3 - 1/b^3} - \frac{\bar{\sigma}_b - \bar{\sigma}_c}{1/c^3 - 1/b^3} \frac{1}{r^3} \qquad \text{(B2)}$$

$$\bar{\sigma}_t = \frac{\bar{\sigma}_b / c^3 - \bar{\sigma}_c / b^3}{1/c^3 - 1/b^3} + \frac{\bar{\sigma}_b - \bar{\sigma}_c}{1/c^3 - 1/b^3} \frac{1}{2r^3} - \frac{1-2\nu}{1-\nu} \frac{Q}{8\pi k} \frac{1}{r} \qquad \text{(B3)}$$

$$u = \frac{1-2\nu}{E} \frac{\bar{\sigma}_b / c^3 - \bar{\sigma}_c / b^3}{1/c^3 - 1/b^3} r + \frac{1+\nu}{2E} \frac{\bar{\sigma}_b - \bar{\sigma}_c}{1/c^3 - 1/b^3} \frac{1}{r^2} - \frac{(1+\nu)(1-2\nu)}{(1-\nu)E} \frac{Q}{8\pi k} \qquad \text{(B4)}$$

References

Adachi, T. (1986). “Geotechnical report on the Seikan tunnel”. *TUST,* **1**: 351-355.

Anagnostou, G. and Kovari, K. (2005). “Tunnelling through geological fault zone”. *Int. Symp. on Design Construction and Operation of Long Tunnels; Taipei, Taiwan*: 509-520.

Bilfinger, W. (2005). “Impermeabilisation versus drainage – Some considerations regarding lining loads”. *Felsbau*, v. **23**: 55–61.

Deere, D.U. and Lombardi, G. (1989). “Lining of pressure tunnels and hydrofracturing potential”. *Victor de Mello Volume, Editora Edgard Blücher Ltda,, São Paulo, Brasil*: 121-128.

Egger, P., Ohnuhki T. and Kanoh, Y. (1982). “Bau des Nakayama-Tunnels Kampf gegen Bergwasser und vulkanisches Lockergestein”. *Rock Mechanics*, Suppl. **12**: 275–293.

Fernandez, G. and Alvarez, J. (1994). “Seepage-Induced Effective Stresses and Water Pressures Around Pressure Tunnels”. *Journal of Geotechnical Engineering*, vol. **120**: 108-128.