Some Comments on the Convergence-Confinement Method

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1. INTRODUCTION

THE GENERAL REPORT just presented shows clearly and completely the principle of the convergence-confinement method and the use that can be made of the different curves or characteristic lines.

I agree with the report, overall, and wish only to emphasize some details and propose some generalizations based on recent studies.

Two basic considerations relative to the terms used should be noted:

- Perhaps it would be better to begin immediately to refer to the method as "confinement-convergence" rather than "convergence-confinement," in order to emphasize the fact that in calculations, confinement is considered to be the independent variable, or principal argument of the problem, with the convergence being determined by the instantaneous value of the confinement.

- Even though the expression "characteristic line" has a specific and different meaning in continuum mechanics, it is necessary nevertheless to use the term here also, to denote the curves that characterize the behavior of the various factors involved in our problem.

In particular we will be concerned with:
- the characteristic lines representing the opening at various distances from the face at different times,
- the characteristic lines for the tunnel at the face of the same heading,
- the characteristic lines for the unexcavated "core" of rock immediately ahead of the face,
- the characteristic lines for the supports, as installed.

All these lines or characteristic curves will vary as a function of time and other factors.

2. THREE-DIMENSIONAL STATE

2.1. General

It is well known that the state of equilibrium at the face of the tunnel is not the same as it is at some distance from the face. We can define three major reasons for this:

1. the resistance to convergence presented by the as yet unexcavated core of rock ahead of the face;
2. deviation of the longitudinal stresses from parallelism with the tunnel axis;
3. the effect of time, i.e., the rheological behavior of the rock.

2.2. Resistance of the unexcavated rock core

If we extend the plane of the face towards the exterior of the tunnel, we can easily see that there are radial shear forces along this plane, acting as indicated in the left-hand side of Fig. 1.

These forces result from the fact that the unexcavated zone is more rigid than that from which the rock has been removed, the latter being supported by the core of rock ahead of the face.

To examine this phenomenon, consider a section of rock perpendicular to the axis of the tunnel immediately at the face.

Let us now assume that half of the section of rock is removed, by advance of the tunnel. As this occurs, this core of rock will either behave elastically, or it may be overstressed so that it behaves plastically. This behavior is illustrated in the diagram on the right-hand side of Fig. 1, using the characteristic lines for the opening and for the core. Line 1 is the characteristic curve for the opening that starts at point A, indicative of the natural stress in the rock mass.

If we could extract the core of rock in front of the face without disturbing it, we would observe that, freed from the stresses acting there, it would unload along characteristic line 2 with radial expansion equal to 0 B. If we could now replace the core, or rather the section corresponding to the core of one-half the original thickness into the excavation it would deform as indicated by characteristic line 3. With the thickness reduced by half, the elastic compliance would in effect be doubled. The equilibrium position of the section under consideration would be at C.
assuing that the core behaves elastically throughout the unloading. If, on the other hand, we note that the state of stress in the half-core is no longer triaxial, since the longitudinal stresses on the face of the excavation are zero, then in the majority of cases the strength of the core will be reached and the characteristic line will change slope to follow curve 4. Thus the equilibrium position is displaced from C to D. In the face, this equilibrium corresponds to the strength limit of the reduced thickness core.

Numerous observations reveal that the strength limit of the face is frequently reached, which is sometimes exhibited by instability of the face and by some form of extrusion into the opening.

The method that I have just outlined is intended to allow determination of deformation O E, which takes place in the plane of the face of the heading, and to ascertain the convergence in the plane of the face at the moment of support installation, if this is placed immediately upon excavation.

I should point out that M. Panet has also presented a similar method, which is based, however, on consideration of the equilibrium of a hemispherical zone near the face of the heading, rather than on that of a planar core.

2.3. Deviation of longitudinal stresses

Another phenomenon that should be considered occurs near the face of the heading. It is too often forgotten in considering the tunnel as a problem of plane equilibrium that the longitudinal stresses, i.e., those parallel to the axis of the tunnel, do indeed exist. They do disappear (unless some special method of construction is used) at the face of the heading itself. This results in a deviation of the longitudinal stress trajectories as shown in Fig. 2.
The curvature of these trajectories gives rise to "deviation forces," which, on the one hand, add to the compression of the core in front of the face and, on the other, support the excavation.

It is possible to estimate the order of magnitude of the effect of these deviation forces, which allows us, as shown in Fig. 2, to calculate a second curve 2 that is valid in the vicinity of the face of the heading, in addition to characteristic curve 1 for the excavation.

2.4. Combination of characteristic lines for the face of the excavation

Taking the above factors into account we can easily construct equilibrium diagrams for the face of the heading, as in Fig. 3.

![FIG. 3. Characteristic lines for the support installed at the face of the excavation.](image)

Characteristic line 1 is that for the excavation near the face. The intersection of this with the characteristic line for the weakened core, 2, leads to the point of equilibrium B, which determines the convergence OC at the moment of installation of the lining (if it is placed right at the face of the heading).

The lining characteristic 4 crosses line 3 at D, which defines the behavior of the opening at some distance from the face. Point D corresponds, therefore, to the equilibrium of the opening at some large distance from the face of the heading. In this case, the deformation OE is the final convergence, obviously greater than the convergence OC in the plane of the face.

3. CONFINING PRESSURE AND THE DECOMPRESSED ZONE

In the General Report, it is hypothesized that the support pressure that must be exerted at the crown of the tunnel will be the sum of two terms—one being the pressure of confinement, which decreases with convergence; and the other the pressure applied to assure stability of the decompressed zone, which increases with convergence.

We offer a slightly different approach to this point. The characteristic line is calculated on the assumption of a continuum. If, in reducing the support pressure, tension forces are generated (e.g., in the roof) that the rock cannot sustain, discontinuities are produced in the stress field, and hence, the characteristic line is not valid beyond this limit. Since such stresses are generated, we must obviously take into account the weight of the rock mass and, as we shall see, this can be done, and in fact has been done readily for a considerable time.

Always, however, even though fractures open and a mass of rock becomes detached, the roof above this mass must be stable. If this overlying roof were not stable, then the discontinuity referred to would not in fact form. Now, there can be only two cases for which the discontinuity will form, namely:

- the curvature of the new roof is greater than that of the opening, which indicates that the roof of the opening is too flat, or
- the rock mass is inhomogeneous, such that distinct discontinuities separate a certain volume of rock from the rock mass, which itself is more resistant. This inhomogeneity can be produced by the method of excavation.

In each case, it is necessary to interrupt the calculated characteristic line and to extend it as a straight (vertical) line, the magnitude of which is determined by the weight of the mass detached from the rock mass, as shown in Fig. 4.

![FIG. 4. The three regions of the characteristic line of an excavation.](image)

The characteristic curve, in fact, consists of three parts:

1. a section due to the elastic deformations between A and B,
2. a section determined by plastic deformations,
from B to C, and where, consequently, shear fractures occur, and

(3) a section from C to D, where brittle fractures develop due to tensions in the roof.

Thus, we can speak of elastic convergence, plastic convergence, and collapse. Depending on the situation, one or another of these segments may not appear, or the characteristic line could even be represented by a single straight line A E — this is the highly theoretical case of perfectly elastic behavior.

This representation of the characteristic line is very much in accord with what is said in the General Report, in the sense that the support pressure hardly varies as a function of the convergence around the minimum represented by the straight line C D.

The same figure shows three different representations of the characteristic line. The first two are the most usual. The third is perhaps preferable since in this case flow is indicated by a downward movement. (General agreement on a single representation would simplify the issue.)

Referring again to the characteristic line for the excavation we recall that it is usually calculated on the assumption that certain rock properties, in particular cohesion and friction, remain constant.

It may happen, following what is called the “decompression” of the rock, that the geotechnical characteristics on which the calculations are based no longer hold, at least in a certain zone of the rock mass. In this case, we can not use the same characteristic curve.

To describe the situation in its complexity, we must then take into account a complete series of characteristic curves each calculated for a reduced value of rock resistance in a zone of greater or lesser extent around the tunnel. These curves, which indicate progressively larger convergences, are in fact each valid only for a narrow zone; in the limit they are valid only at one point (a single radius). Passage from one to the other depends on the rigidity of the lining. This phenomenon is explained in Fig. 5.

The geometric locus of all of these “points of validity” can, if one wishes, also be defined as a characteristic curve, such as the section C D in Fig. 5, which considers a decompressed zone of increasing extent. We see that this exhibits the phenomenon that is often interpreted as a “climbing back up” of the characteristic line.

In reality, the case of a mass that is detached from the roof should be combined with the characteristic line, in the manner discussed earlier. Thus, the weight of this mass is substituted for the confinement pressure, but not added to it.

These phenomena, which may seem complex, can be explained quite simply if the effect of gravity is included from the beginning in the calculation of the rupture zone, instead of trying to introduce it somewhat artificially at the end of the calculation. Figure 6, which dates from 1968, shows the results of such calculations. The extension of the rupture zone is asymmetrical with respect to the profile of the tunnel, precisely because gravity was taken into consideration from the beginning.

Figures 7 and 8 show equally well the effect of gravity on the stress and deformation fields in the case of an underground hydroelectric installation, now under construction.

4. GENERALIZATION OF THE CONCEPT OF CHARACTERISTIC LINES

The General Report correctly presents the concept of the characteristic line for the excavation for a simple case with quite restrictive hypotheses. It is essentially the case of axial symmetry calculated by neglecting the effect of gravity. These limitations are not appropriate for the methods of concern to us, but obviously facilitate explanation and calculation.

We have just seen that gravity can be taken into account in calculation of the fracture zones. Furthermore, it is possible to generalize the idea of the characteristic line by defining it as the “geometric locus of possible equilibrium conditions.” The convergence, and also the confinement, are no longer scalar quantities, but, because of their various components, may be considered as vectors, which should be drawn in multi-dimensional space as characteristic lines rather than as characteristic surfaces having several dimensions.

In practice however, it is often possible to use a single parameter such as confinement or convergence, and to be satisfied, as before, with a planar representation.

If, for example, we expect to support a noncircular excavation by arches, we are well aware that the reactions applied by the arch to the rock vary, to a good approximation, as the curvature of the arch. This pressure should therefore be a

FIG. 5. Characteristic lines as a function of the extent of the decompressed zone.
FIG. 6. Extent of the zone of limiting equilibrium as a function of the confining pressure.

FIG. 7. Central power station – Albi; zone of failure and stress-field.

FIG. 8. Central power station – Albi, Stabilisation pressure and stress-field.
maximum in the crown of the roof; diminish towards the sides; and become zero along these, if they are straight sides.

Knowledge of either the pressure in the crown or the axial force in the arch support is sufficient to characterize the ensemble of reactions that the former exert on the edge of the excavation. Thus, we can use this single parameter (pressure or axial force) as an argument of the characteristic line.

So far as deformation is concerned, it is not necessary to study convergence movements at all points of the excavation perimeter. Instead, we may consider the shortening of the excavation perimeter, which alone adequately characterizes the totality of convergence movement since it is an integral value of sorts. We could then select the shortening of the rock perimeter as a given value and set the shortening of the support axis to correspond to this.

As soon as we define parameters adequate to represent confinement and convergence, we should be able to develop the characteristics in a new way even for asymmetrical cases, for excavations of any form, and for inhomogeneous stress conditions. Already, we see that we can easily take account of gravity. In doing this, we generalize the concept of the characteristic line; we give it the sense of “locus of possible equilibrium conditions.” In this way, it becomes an intuitive representation of phenomena that affect the stability of the excavation. (Figure 9 shows this generalization of the characteristic line concept.)

Under these conditions, we are obviously no longer concerned with analytical calculations, but must resort to numerical methods, which, in principle, pose no obstacles. The numerical methods that can be used are the well-known finite-element method, as well as others that are more precise or significantly more efficient.

By contrast, Fig. 10 concerns a specific case of excavation of an opening in several stages. Here we have taken as the convergence parameter the volume of rock that “crosses” the theoretical contour of the tunnel per meter of length. This is, in effect, an integral value of the radial deformation over the entire periphery.

For the abscissa, i.e., representing confinement, we have for example for the upper line the values of the reaction pressure in the arch support at the two points A and B, which are proportional to the normal force N in the support.

5. RHEOLOGICAL MODEL OF THE ROCK MASS

The question raised by the general reporter as to the choice of rheological model to use cannot, in my opinion, be answered in general.

The particular rheological model to use will depend on the nature of the rock and type of discontinuities. Only through detailed study of the characteristics of the terrain concerned, together with measurements of actual behavior of the excavation, can one best approach the construction of realistic models.

Using this approach, we have been able to demonstrate that a rheological model composed of two Bingham bodies and of one elastic (Hooke) element can give satisfactory results for certain crystalline rocks. The model is shown in Fig. 11. Each Bingham body is composed of a Saint-Venant element that simulates friction, and a Newton element to represent viscosity. We can add to these a rupture element in order to represent the “peak” or maximum shear resistance that is observed in certain rocks.

This rheological model has been used to calculate the convergences and to determine the confinements in the so-called “Mesozoic” section of the St. Gothard highway tunnel, and has given satisfactory results, as shown in Figs 12 and 13.

There is no doubt that the so-called “convergence-confinement” or “characteristic-line” method is very useful in judging the behavior of underground openings. It represents a particularly simple method that requires only limited calculations. One can confirm its validity through more profound studies.

Along these lines, we have formulated what may
FIG. 10. St Gotthard tunnel – Mesozoic zone – characteristic lines for different phases of construction.

FIG. 11. Rheological model.
be called a "model for the tunnel." This consists of studying the stress history in a given section perpendicular to the axis of the tunnel, as a function of tunnel face advance.

Consider first of all, the equilibrium conditions for the rock core ahead of the excavation as indicated in Fig. 14. At a certain distance ahead of the face, the core is subjected to the natural state of stress in the terrain. Gradually, as the distance from the face is reduced, the state of triaxial stress tends more and more towards a state of two-dimensional stress, which in fact exists at the face itself. The compressional resistance of the core, so weakened, is determined by the plastic properties of the rock; its progressive reduction is indicated in Fig. 14.

Immediately behind the face, the opening is no longer supported from within but, due to the shearing forces that exist between the various sections, equilibrium is maintained until the lining is introduced, the load on it then increasing progressively with face advance, from the point at which the radial gap due to installation has closed until final equilibrium is attained at a substantial distance behind the face. This calculation is made assuming axial symmetry and some simplifications as to the stress distribution in the planes perpendicular to the tunnel axis. It can be modified to allow for variation in the duration of the pressure, acting first on the core and then on the lining, as well as convergence movements ahead of and behind the face.

The relation between the two values, i.e., the confinement and the convergence, can be established at each instant. These relations are displayed in Fig. 15 which shows the well-known characteristic lines for the face, the unexcavated "core," the opening, and the lining. For the opening, we distinguish the early characteristic line from that which obtains at

FIG. 14. Model for the tunnel.
final equilibrium. The actual convergence-confinement relation is indicated by a line of arrows, with time given in parametric form. The geomechanical characteristics used in the calculation are given on Fig. 15, as is the rigidity of the lining.

briefly to some of the questions raised by the organizers of this meeting, I would like to conclude as follows:

The so-called "convergence-confinement" method uses many curves that characterize the behavior of various elements that enter into the problem. We can very well call them curves or characteristic lines.

The restrictive hypotheses that are sometimes made -- for example, those relative to axial symmetry or to gravity -- are not implicit in the method, but allow a simplification that is sometimes useful.

The method in question can be used practically for study of a project, but can be employed equally well to interpret the convergences observed around the working face.

The principal parameters necessary to determine the curve for the ground are the following:

CONCLUSIONS

Summarizing what I have said and responding
• the natural state of stress before construction,
• the deformability characteristics of the ground in its elastic, plastic, and viscous phases,
• values for the peak and residual angles of friction and cohesion,
• dilatation at the onset of irreversible deformation.

The values of the viscosity and dilatation parameters are certainly the most difficult to obtain.

As a general conclusion I would like to say that, as with all calculations, the convergence-confinement method cannot give results that are more exact than the assumptions on which it is based.

Thus, the precision is limited by the geotechnical values used and, let us not forget, by the extent to which the anticipated method of working is actually followed during excavation and installation of the linings.

It is obvious that there is interest, on the one hand, to go ahead using the information that is available, recognizing the uncertainties; and, on the other, to strive to obtain, in time and with the required precision, all the necessary basic parameters.

The fact that it is essential to verify the behavior of the ground and support under actual working conditions does not, I think, need any particular emphasis here.